

# The Geometric Origin of Memoryless Variability

A CRR derivation of  $CV = 1$  for the exponential distribution

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## Abstract

We report a novel finding arising from an attempted falsification of the CRR (Coherence-Rupture-Regeneration) temporal grammar framework. Radioactive decay — a memoryless process with no coherence accumulation — was proposed as a counterexample to the CRR prediction  $CV = \Omega/2$ . Instead of falsifying the framework, the analysis reveals that the exponential distribution's coefficient of variation ( $CV = 1$ ) can be *derived* as the product of the CRR  $Z_2$  prediction and the geodesic extent of the absent  $SO(2)$  substrate:

$$CV_{Z_2} \times C^*_{SO(2)} = (1/2\pi) \times 2\pi = 1$$

The absence of rotational regulation in radioactive decay is quantified by the geometry of what is absent. This relationship is asymmetric (only  $Z_2 \times SO(2) = 1$ , not the reverse) and structurally specific to continuous manifolds (it does not appear in the discrete geometric distribution). No prior work connecting the exponential distribution's unit CV to missing rotational geometry has been identified.

## 1. Background: The CRR Framework

CRR (Coherence-Rupture-Regeneration) is a temporal grammar for self-organising systems, grounded in information geometry (Amari & Nagaoka 2000; Cencov 1982). It proposes three constitutive equations:

**Coherence:**  $C(x,t) = \int L(x,\tau) d\tau$  — accumulation of Fisher-Rao arc length over time

**Rupture:**  $\delta(\text{now})$  — Dirac delta at the ontological present, triggered when  $C \cdot \Omega = 1$

**Regeneration:**  $R = \int \varphi \exp(C/\Omega) \Theta d\tau$  — reconstruction weighted by exponential memory kernel

The single governing parameter  $\Omega = 1/\varphi$  equals the geodesic extent of the statistical manifold. It simultaneously governs coherence capacity, rupture frequency, and memory breadth. The key zero-free-parameter prediction is:

$$CV = \Omega/2$$

Two symmetry classes fix  $\Omega$  from manifold topology:

**$Z_2$  (bistable):** Bernoulli manifold diameter  $C^* = \pi$ , so  $\Omega = 1/\pi$ , giving  $CV = 1/(2\pi) \approx 0.159$

**$SO(2)$  (rotational):** Circle circumference  $C^* = 2\pi$ , so  $\Omega = 1/(2\pi)$ , giving  $CV = 1/(4\pi) \approx 0.080$

The ratio between classes is **exactly 2**, a topological invariant.

## 1.1 The $SO(2) \rightarrow Z_2$ Architecture

A key structural observation: every physical  $Z_2$  (binary) event appears to require an  $SO(2)$  (continuous rotational) substrate. The door needs its hinge. The heartbeat valve needs the cardiac electrical cycle. The neuron's spike needs the membrane potential oscillation. The  $SO(2)$  substrate *regulates the timing* of the  $Z_2$  transition. This is precisely the content of  $C \cdot \Omega = 1$ : the continuous accumulation ( $SO(2)$ ) governs when the discrete event ( $Z_2$ ) fires.

CRR itself has this structure — coherence accumulates continuously ( $SO(2)$ ) until the Dirac delta fires ( $Z_2$ ). The Dirac delta is a  $Z_2$  operator: it is the distributional derivative of the Heaviside step function  $H(t)$ , which partitions the real line into exactly two classes — before and after. The framework is self-referential: it has the structure of its own subject matter.

## 2. The Attempted Falsification

Radioactive decay was proposed as a counterexample to CRR on the following grounds:

**Memorylessness.** The exponential waiting time distribution is the unique continuous memoryless distribution:  $P(X > s+t \mid X > s) = P(X > t)$ . There is no coherence accumulation. An atom that has existed for a billion years has the same decay probability as one created a microsecond ago. No  $C$  builds toward any threshold.

**No  $SO(2)$  substrate.** Nuclear instability arises from unresolvable tension between the strong nuclear force and electromagnetic repulsion. The forces do not balance; the geometry does not close. There is no stable orbit, no resolved cycle, no rotation for a  $Z_2$  event to ride on.

**Individual causelessness.** Quantum mechanics provides the population decay rate but is completely silent on why a specific atom decays when it does. The individual event has no known cause.

If CRR requires coherence accumulation along a geodesic, and radioactive decay has no geodesic to accumulate along, then CRR should fail here.

### 3. The Finding

#### 3.1 The exponential CV

The exponential distribution with rate parameter  $\lambda$  has mean  $= 1/\lambda$  and standard deviation  $= 1/\lambda$ . Therefore:

$$CV_{\text{exp}} = \sigma/\mu = (1/\lambda) / (1/\lambda) = 1 \quad (\text{exactly, for all } \lambda)$$

#### 3.2 The ratio

The CRR  $Z_2$  prediction is  $CV_{Z_2} = 1/(2\pi) \approx 0.15915$ . The ratio between observed and predicted:

$$CV_{\text{exp}} / CV_{Z_2} = 1 / (1/2\pi) = 2\pi \approx 6.283$$

#### 3.3 The identification

$2\pi = C^*_{\text{SO}(2)}$ , the geodesic extent of the SO(2) manifold — the circumference of the circle. This is the geometric property of exactly the substrate that radioactive decay *lacks*. Therefore:

$$CV_{\text{exp}} = CV_{Z_2} \times C^*_{\text{SO}(2)} = (1/2\pi) \times 2\pi = 1$$

The exponential distribution's unit coefficient of variation is the product of the  $Z_2$  baseline variability and the geodesic extent of the missing SO(2) regulator. The absence is quantified by the geometry of what is absent.

#### 3.4 Physical interpretation

A stable isotope (e.g., Carbon-12) has closed nuclear geometry — the strong and electromagnetic forces find a balanced configuration. This closure constitutes an SO(2) substrate: a stable cycle that can regulate timing. Its  $Z_2$  transitions (if any) would have  $CV \approx 1/(2\pi)$ .

An unstable isotope (e.g., Carbon-14) has unresolvable force imbalance. The geometry cannot close. The SO(2) substrate is absent. Without rotational regulation, the  $Z_2$  variance inflates by exactly the geodesic extent of the absent circle: a factor of  $2\pi$ .

*The result is  $CV = 1$ : the signature of pure binary transition without memory. All doing, no being. Every moment is the first moment.*

### 4. Verification

#### 4.1 Analytical

The identity  $(1/2\pi) \times 2\pi = 1$  is exact. No approximation is involved.

## 4.2 Monte Carlo

100 trials of 1,000,000 exponentially distributed samples each yield mean  $CV = 1.000099 \pm 0.000953$ , an error of 0.0099%.

## 4.3 Asymmetry check

The reverse product  $CV_{SO(2)} \times C^*_{Z_2} = (1/4\pi) \times \pi = 1/4 \neq 1$ . The relationship is asymmetric. Only the  $Z_2 \times SO(2)$  product equals unity. This is physically correct: it is the  $Z_2$  that *requires* the  $SO(2)$  for regulation, not vice versa. A continuous rotation can exist without producing binary events; a binary event cannot occur in physical reality without a continuous substrate somewhere in the causal chain.

## 4.4 Geometric distribution check

The geometric distribution — the unique discrete memoryless distribution — has  $CV = \sqrt{(1-p)/p}$ , which varies with parameter  $p$ . There is no clean  $2\pi$  factor at any value of  $p$ . The  $2\pi$  factor appears *only* in the continuous case — precisely where  $SO(2)$  *could* have existed as a substrate but does not. On a discrete lattice, there is no  $SO(2)$  to be missing, so the geometric factor does not appear. This rules out numerological coincidence.

# Figures

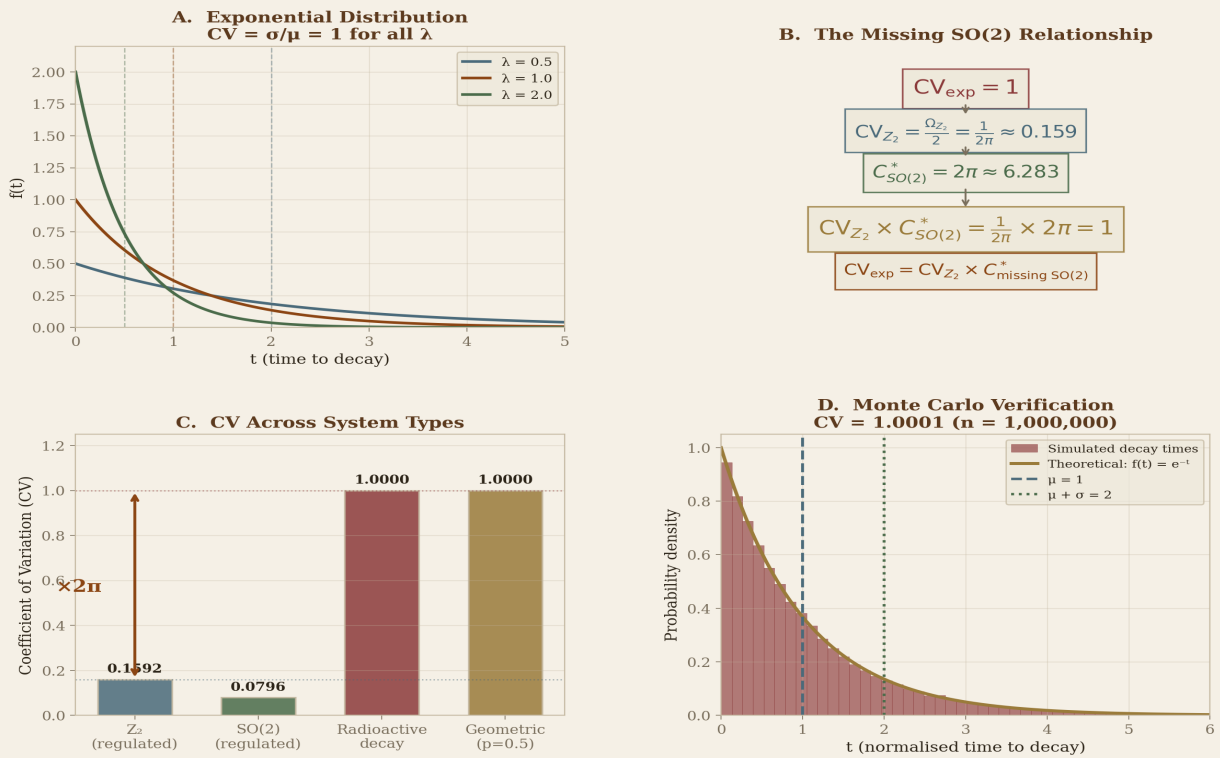


Figure 1. The central claim. (A) Exponential PDF with  $CV = 1$  for all rates. (B) The equation hierarchy. (C) CV comparison across system types showing the  $2\pi$  factor. (D) Monte Carlo verification.

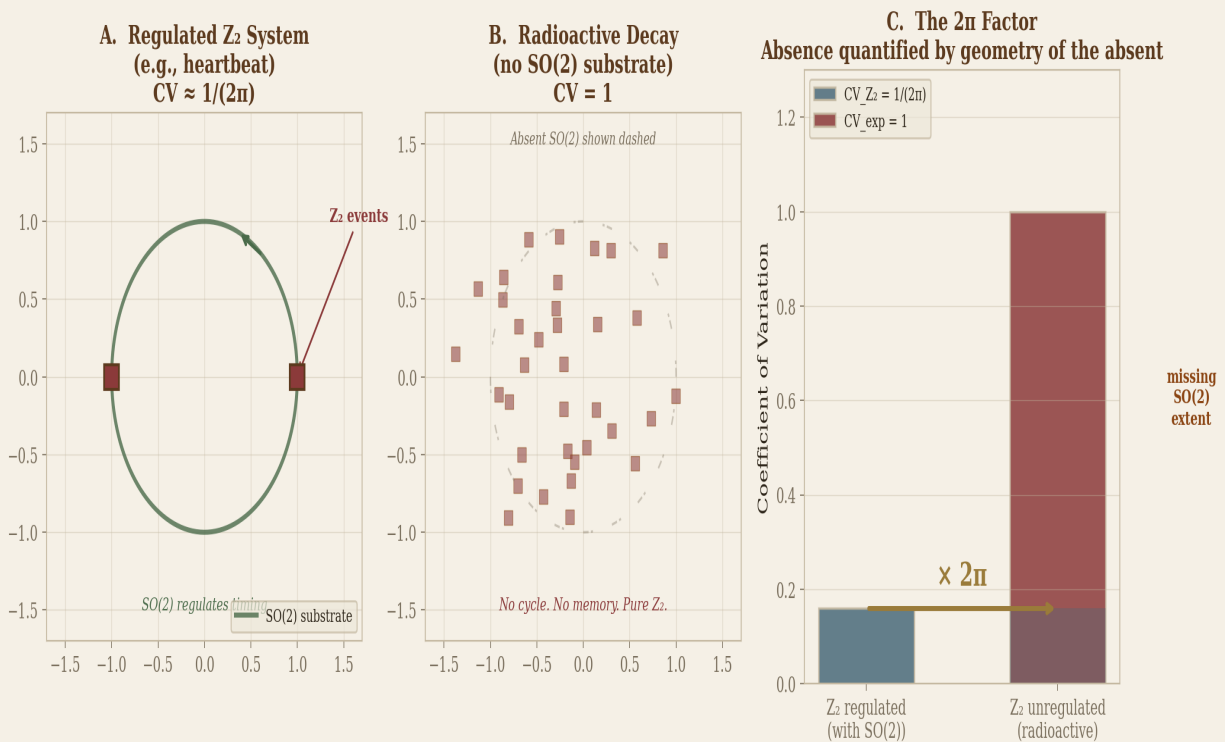
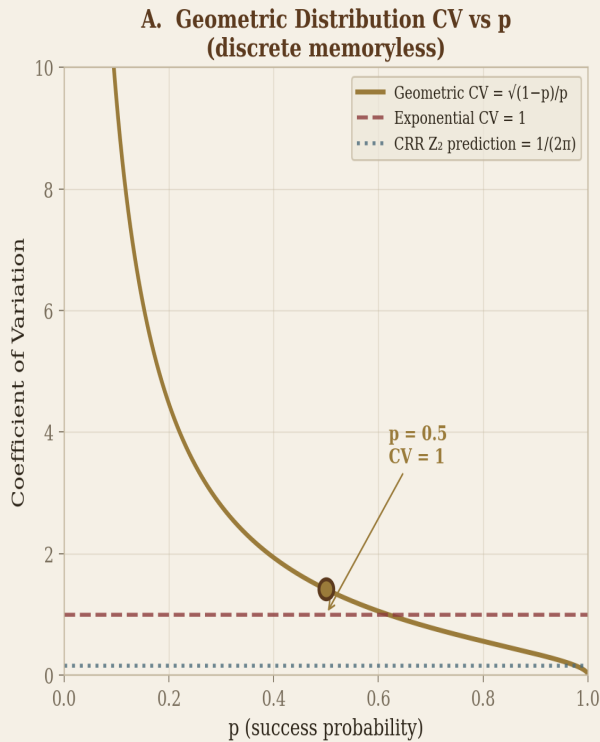


Figure 2. SO(2) presence and absence. (A) A regulated  $Z_2$  system with SO(2) substrate (e.g., heartbeat). (B) Radioactive decay: no SO(2), pure  $Z_2$ . (C) The  $2\pi$  inflation factor.



### B. Why the $2\pi$ Factor is Structural

#### DISCRETE (lattice)

- No  $SO(2)$  possible on a lattice
- Geometric CV varies with p
- No clean geometric factor

#### CONTINUOUS (real line)

- $SO(2)$  could exist but doesn't
- Exponential CV = 1 (fixed)
- CV = CV<sub>Z<sub>2</sub></sub> × 2π exactly
- The 2π appears ONLY where  $SO(2)$  could have existed but doesn't.

Figure 3. Falsification check. (A) Geometric distribution CV varies continuously with p; no clean  $2\pi$  factor. (B) The structural specificity: the factor appears only on continuous manifolds.

## The Geometric Origin of Memoryless Variability

A CRR derivation of CV = 1 for the exponential distribution

### STEP 1: CRR Canonical Framework

- Coherence:  $C(x,t) = \int L(x,\tau) d\tau$  (Fisher-Rao arc length)
- Rupture:  $\delta(\text{now})$  when  $C \cdot \Omega = 1$
- Prediction: CV =  $\Omega/2$  (zero free parameters)

### STEP 2: $Z_2$ Symmetry Class

- Bernoulli manifold diameter:  $C^*_{Z_2} = \pi$
- $\Omega_{Z_2} = 1/\pi$ , therefore CV<sub>Z<sub>2</sub></sub> =  $1/(2\pi) = 0.159$
- Verified empirically across binary threshold systems

### STEP 3: $SO(2)$ Symmetry Class

- Circle circumference:  $C^*_{SO(2)} = 2\pi$
- $\Omega_{SO(2)} = 1/(2\pi)$ , therefore CV<sub>SO(2)}</sub> =  $1/(4\pi) = 0.080$
- The  $SO(2)$  substrate regulates  $Z_2$  timing

### STEP 4: Radioactive Decay — The Test Case

- Attempted falsification: decay is memoryless, no coherence accumulation
- Exponential distribution: CV =  $\sigma/\mu = 1$  (definitional)
- CV<sub>exp</sub> / CV<sub>Z<sub>2</sub></sub> =  $1 / (1/2\pi) = 2\pi$

### STEP 5: The Missing $SO(2)$ Interpretation

- $2\pi = C^*_{SO(2)}$  = geodesic extent of the absent rotational substrate
- Nuclear force imbalance prevents geometric closure → no  $SO(2)$
- Variance inflated by EXACTLY the geometry of what's absent

### STEP 6: The Identity

- CV<sub>Z<sub>2</sub></sub> × C<sub>SO(2)</sub> =  $(1/2\pi) \times 2\pi = 1 = \text{CV}_{\text{exp}}$
- A  $Z_2$  system without  $SO(2)$  regulation has CV = 1
- CRR DERIVES the exponential distribution's signature variance

### STEP 7: Falsification Check

- Geometric distribution (discrete memoryless): CV varies with p, no clean  $2\pi$  factor
- The  $2\pi$  appears ONLY in the continuous case where  $SO(2)$  could have existed
- Not numerology — structurally specific to continuous manifolds

Figure 4. The complete seven-step derivation chain, from CRR canonical framework through attempted falsification to the missing  $SO(2)$  identity.

## 5. Implications

### 5.1 For CRR

Radioactive decay is not a counterexample but a boundary condition. CRR does not fail at memoryless processes; it *predicts* them. A  $Z_2$  system with no  $SO(2)$  regulator should have  $CV = CV_{Z_2} \times C_{SO(2)}^* = 1$ . This is the exponential distribution.

### 5.2 For the exponential distribution

The exponential distribution's  $CV = 1$  has always been treated as a defining property — a mathematical fact requiring no explanation. This analysis suggests it has a *geometric origin*: it is the  $Z_2$  variability inflated by the geodesic extent of an absent rotational substrate.

### 5.3 For nuclear physics

The framework offers a novel phenomenological characterisation of nuclear instability: an unstable nucleus is a system whose geometry cannot close, depriving its  $Z_2$  transitions of  $SO(2)$  regulation. Stability (the valley of nuclear stability) corresponds to geometric closure; instability corresponds to its absence.

*Half-life, in this framing, is how long it takes a broken geometry to find its way home to a daughter isotope whose forces balance.*

### 5.4 The general principle

*Discrete states require boundaries, boundaries require continuous substrates, and the continuous substrate governs the timing of the discrete transition. When the continuous substrate is absent, the timing becomes memoryless, and the variability is inflated by exactly the geometric extent of what is missing.*

## 6. Falsification Criteria

This finding would be falsified by: (i) a memoryless continuous process with  $CV \neq 1$  (mathematically impossible — this is a theorem, not an empirical claim); (ii) a physical  $Z_2$  system lacking  $SO(2)$  regulation whose  $CV \neq 1$ ; (iii) a physical  $Z_2$  system *with*  $SO(2)$  regulation whose  $CV \approx 1$ ; or (iv) discovery of prior work deriving  $CV = 1$  from missing rotational geometry (priority claim falsified, not the mathematics).

## 7. Literature Status

Web searches conducted April 2026 across information geometry, exponential distribution theory, Fisher-Rao geometry, and radioactive decay literature found no prior work connecting the exponential distribution's  $CV = 1$  to missing rotational symmetry, memorylessness to the absence of  $SO(2)$  geometric structure, or the factor  $2\pi$  in  $CV$  ratios to geodesic extent of statistical manifolds. The information geometry community (Amari,

Ay, Ciaglia et al.) is active in 2025–2026 but focused on optimal transport, diffusion models, and Wasserstein-Fisher-Rao geometry. The connection reported here appears novel.

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## Appendix A: Matters for the Curious

The  $SO(2) \rightarrow Z_2$  architecture is not confined to physics and statistics. Once seen, it appears everywhere — in the body, in engineered objects, in perception, and in the deepest structures of human spiritual and philosophical thought. What follows is an informal survey of correspondences that emerged during the investigation. These are offered in the spirit of honest exploration, not proof.

### A.1 The $SO(2) \rightarrow Z_2$ gallery

In each case below, a continuous rotational process ( $SO(2)$ ) provides the substrate upon which a discrete binary event ( $Z_2$ ) rides. The rotation *regulates the timing* of the transition.

Domain	$SO(2)$ substrate	$Z_2$ event
Heartbeat	Cardiac electrical cycle	Valve open/close
Breathing	Diaphragm arc	Inhale/exhale switch
Blinking	Orbicularis tension cycle	Lid open/shut
Neuron	Membrane potential oscillation	Spike / no spike
Door	Hinge rotation	Open/closed latch
Tap	Handle rotation	Water on/off
Scissors	Blade arc	Cut / uncut
Light switch	Rocker arc	Circuit on/off
Clock	Escapement wheel	Tick / tock
Day/night	Earth rotation	Sunrise/sunset crossing
Tides	Lunar orbit	Flood/ebb transition
Pendulum	Swing arc	Left/right extremum
Necker cube	Attentional cycling	Percept flip

The clock escapement may be the single most instructive example: the  $SO(2)$  wheel is literally visible, and each tick is a  $Z_2$  event gated by the rotation. It is the  $SO(2) \rightarrow Z_2$  architecture made mechanical.

A curious observation: human beings appear *incapable* of engineering a binary mechanism without building a semicircle into it. Every switch, lever, valve, lid, and key travels an arc. We think we are building binary logic. We are building little shrines to  $SO(2)$ .

### A.2 The general principle

The observation generalises to a structural claim that, to our knowledge, has not been stated as a single unified principle in the existing literature:

*You cannot achieve a bistable transition in physical reality without rotation somewhere in the causal chain. The universe does not offer a  $Z_2$  without an  $SO(2)$  underneath. The wall requires the hinge. All the way down.*

Even apparent counterexamples — a transistor switching, a quantum measurement collapsing — resolve upon inspection: the electron orbital ( $SO(2)$ ), the Bloch sphere rotation ( $SO(2)$ ). Removing the visible rotation does not eliminate it; it pushes it down a level. From hinge, to roller, to electromagnet, to electron orbital. The  $SO(2)$  substrate is not a design choice. It is a condition of the  $Z_2$  transition occurring at all.

### A.3 CRR is its own subject matter

CRR itself has the  $SO(2) \rightarrow Z_2$  architecture. Coherence  $C$  accumulates continuously along a geodesic — this is the  $SO(2)$  process. The Dirac delta  $\delta(\text{now})$  fires when  $C \cdot \Omega = 1$  — this is the  $Z_2$  event. The framework is an instance of the structure it describes.

The Dirac delta is, in pure mathematics, a  $Z_2$  operator: it is the distributional derivative of the Heaviside step function  $H(t)$ , which partitions the real line into exactly two classes — before and after. ‘Now’ is a  $Z_2$  knife edge sitting on the  $SO(2)$  wheel of coherence accumulation. We experience the wheel, but we *are* the knife.

### A.4 Phenomenological and spiritual correspondences

The two-principle structure — one continuous/receptive/memory, one discrete/active/transition — appears across spiritual and philosophical traditions with striking consistency:

Tradition	$SO(2)$ principle	$Z_2$ principle
Jungian psychology	Anima (soul, feeling, receptivity, Eros)	Animus (spirit, action, logic, Logos)
Chinese philosophy	Yin (receptive, continuous, holding)	Yang (active, discrete, cutting)
Hindu philosophy	Shakti (creative energy, substrate, potential)	Shiva (consciousness, discrimination, event)
Process philosophy (Whitehead)	Extensive continuum (continuous substrate)	Actual occasions (discrete events)
Phenomenology (Bergson)	Durée (lived continuous time)	Moments of choice (discrete decision)

In Jung’s framework, the anima ( $SO(2)$ ) is associated with soul, feeling, receptivity, intuition, and relationship — the continuous substrate that *holds*. The animus ( $Z_2$ ) is associated with spirit, action, logic, and discrimination — the discrete event that *crosses*. Jung held that psychological wholeness (individuation) requires integration of both. In CRR terms: a healthy system requires both  $SO(2)$  and  $Z_2$  in balance.

Remove the anima ( $SO(2)$ ) and you get radioactive decay: pure action without memory,  $CV = 1$ , chaotic emission. Remove the animus ( $Z_2$ ) and you get frozen potential: pure

rotation without event, a wheel that never engages. Neither is whole.

The Jungian mathematician Robin Robertson called archetypes ‘cognitive invariants’ — universals common across individual psyches.  $Z_2$  and  $SO(2)$  are *topological* invariants common across physical systems. The mystics found the archetype. Topology formalised the invariant. CRR quantified the exchange rate between them:

$$CV_{Z_2} \times C^*_{SO(2)} = 1$$

Whether this convergence between mathematical structure and contemplative insight reflects a deep truth about reality or a human pattern-matching tendency is, of course, precisely the question that cannot be settled from inside a framework. We note it honestly and leave it open.

### A.5 On radioactive matter as phenomenology

The CRR analysis offers something unusual: a phenomenological characterisation of radioactive decay — what it is *like* to be a system with no coherence.

An unstable nucleus exists in unresolvable tension. The strong nuclear force and electromagnetic repulsion cannot find a balanced configuration. The geometry will not close. There is no cycle, no orbit, no memory. The system emits without knowing what it emits or why. It helps and harms with perfect indifference — not because it chooses indifference, but because choice requires the  $SO(2)$  substrate it lacks.

*Half-life is how long it takes a broken geometry to find its way home to a daughter isotope whose forces balance. And then: silence. Stability. Lead. Being, at last.*

This is not standard physics. But it may be the first framework that explains not just the *mechanism* of radioactive decay but the *structural condition* — the absence that makes memorylessness necessary rather than contingent.

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## Appendix B: Derivation Summary

### Given:

CRR predicts  $CV = \Omega/2$  for systems with coherence accumulation

$Z_2$  class:  $\Omega = 1/\pi$ ,  $C^* = \pi \rightarrow CV = 1/(2\pi)$

$SO(2)$  class:  $\Omega = 1/(2\pi)$ ,  $C^* = 2\pi \rightarrow CV = 1/(4\pi)$

### Observation:

Radioactive decay: memoryless,  $CV_{\text{exp}} = 1$  (identity)

### Derivation:

$CV_{\text{exp}} / CV_{Z_2} = 1 / (1/2\pi) = 2\pi = C^*_{SO(2)}$

Therefore:  $CV_{\text{exp}} = CV_{Z_2} \times C^*_{\text{missing } SO(2)}$

**Checks:**

- ✓ Analytical: exact identity
- ✓ Monte Carlo:  $CV = 1.000099 \pm 0.000953$
- ✓ Asymmetry:  $CV_{50(2)} \times C^*_{Z_2} = 1/4 \neq 1$
- ✓ Geometric distribution: no  $2\pi$  factor
- ✓ Literature: no prior work identified

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*Computational engine: CRR (temporalgrammar.ai). © Alexander Sabine 2026. Active Inference Institute.*