The Journey from A to B:

A Mathematical Analysis of Goal-Directed Behavior Through Free Energy Principle and CRR Frameworks

CRR Research Analysis

January 2026

Abstract

We present a detailed mathematical analysis of a complete driving journey from location A to location B, expressed simultaneously in the Free Energy Principle (FEP) / Active Inference framework and the Coherence-Rupture-Regeneration (CRR) framework. We demonstrate that CRR provides the *dynamical grammar* specifying *when* and *how* the belief updates of active inference occur. The journey is decomposed into nested hierarchical goals, each exhibiting the characteristic pattern: coherence accumulation toward a goal state (prediction), rupture when inside matches outside (prediction confirmation/violation), and regeneration that either reinforces or rewires the agent's model. We show that the universal threshold $\Omega \approx 16$ nats represents the information capacity required to specify a goal with sufficient precision for successful action.

Contents

1 Introduction: Two Frameworks, One Process

Consider the seemingly simple act of driving from your home (A) to your workplace (B). This journey involves hundreds of nested decisions, predictions, actions, and updates—from the macro-level navigation to micro-level muscle contractions. We analyze this through two complementary mathematical frameworks:

- **Free Energy Principle (FEP)**: The brain minimizes variational free energy, which bounds surprise. Action and perception work together to bring predictions and observations into alignment.
- **Coherence-Rupture-Regeneration (CRR)**: Information coherence accumulates until reaching a threshold Ω, triggering rupture and subsequent regeneration that updates the system's state.

Inside-Outside Match 1.1: The Central Insight

The CRR framework provides the **dynamical grammar** of active inference—it specifies:

1. **When** belief updates occur (at rupture, when $C = \Omega$)
2. **How much** historical context influences the update (via $\exp(C/\Omega)$ weighting)
3. **What form** the update takes (regeneration as Bayesian posterior update)

The "inside-outside match" occurs precisely at rupture: the moment when accumulated evidence (C) reaches the precision of the goal specification (Ω).

2 Mathematical Foundations

2.1 The Free Energy Principle

Under the FEP, an agent maintains a generative model $p(o, s)$ of observations o given hidden states s. The agent approximates the true posterior $p(s|o)$ with a recognition density $q(s)$.

FEP Formulation 2.1: Variational Free Energy

$$F = \underbrace{\mathbb{E}_q[\ln q(s) - \ln p(s)]}_{\text{Complexity}} - \underbrace{\mathbb{E}_q[\ln p(o|s)]}_{\text{Accuracy}} \tag{1}$$

Equivalently:

$$F = D_{KL}[q(s)\|p(s|o)] - \ln p(o) \geq -\ln p(o) = \text{Surprise} \tag{2}$$

The agent minimizes F through:

- **Perception**: Update $q(s)$ to better match observations
- **Action**: Change observations o to match predictions

For active inference with goals, we introduce preferences $p(o|\mathcal{G})$ where $\mathcal{G}$ represents the goal:

FEP Formulation 2.2: Expected Free Energy (Goal-Directed)

$$G(\pi) = \underbrace{\mathbb{E}_{\tilde{q}}[D_{KL}[q(s|\pi)\|q(s|o,\pi)]]}_{\text{Epistemic value (information gain)}} + \underbrace{\mathbb{E}_{\tilde{q}}[D_{KL}[q(o|\pi)\|p(o|\mathcal{G})]]}_{\text{Pragmatic value (goal achievement)}} \tag{3}$$

where π is a policy (sequence of actions) and $\tilde{q}$ is the predictive distribution.

2.2 The CRR Framework

The CRR framework describes how systems accumulate coherent information until a phase transition.

CRR Formulation 2.1: Core Definitions

Coherence accumulates evidence toward a goal:

$$C(t) = \int_0^t L(x(\tau), \tau)\, d\tau \tag{4}$$

where L is the coherence density (related to log-likelihood or prediction confirmation).
Rupture occurs when coherence reaches threshold:

$$\text{Rupture at } t^* \text{ when } C(t^*) = \Omega \tag{5}$$

Regeneration updates the system state:

$$R[\phi](t) = \int_{-\infty}^t \phi(\tau) \cdot \exp\left(\frac{C(\tau)}{\Omega}\right) \cdot \Theta(t-\tau)\, d\tau \tag{6}$$

2.3 The Correspondence

We now establish the mathematical correspondence between frameworks:

Concept	**FEP/Active Inference**	**CRR**
Goal state	Prior preference $p(o\|\mathcal{G})$	Threshold Ω
Evidence accumulation	$-\ln p(o\|s)$ integration	Coherence $C(t)$
Prediction error	$\varepsilon = o - g(\mu)$	$dC/dt = L(x,t)$
Inside-outside match	$F \to F_{\min}$	$C \to \Omega$
Belief update	Posterior $q(s\|o)$	Regeneration $R[\phi]$
Precision weighting	$\Pi = \partial^2 F/\partial\mu^2$	$\exp(C/\Omega)$

Table 1: Mathematical correspondence between FEP and CRR frameworks

Inside-Outside Match 2.1: The Key Equation

At rupture ($C = \Omega$), the CRR regeneration weight equals Euler's number:

$$\exp\left(\frac{C}{\Omega}\right)\bigg|_{C=\Omega} = e \approx 2.718 \tag{7}$$

This corresponds to the FEP condition where free energy reaches its minimum for the current goal:

$$\frac{\partial F}{\partial q} = 0 \quad \Leftrightarrow \quad q(s) = p(s|o) \tag{8}$$

The "e-fold" weighting represents optimal Bayesian integration of historical evidence.

3 The Driving Journey: Hierarchical Goal Structure

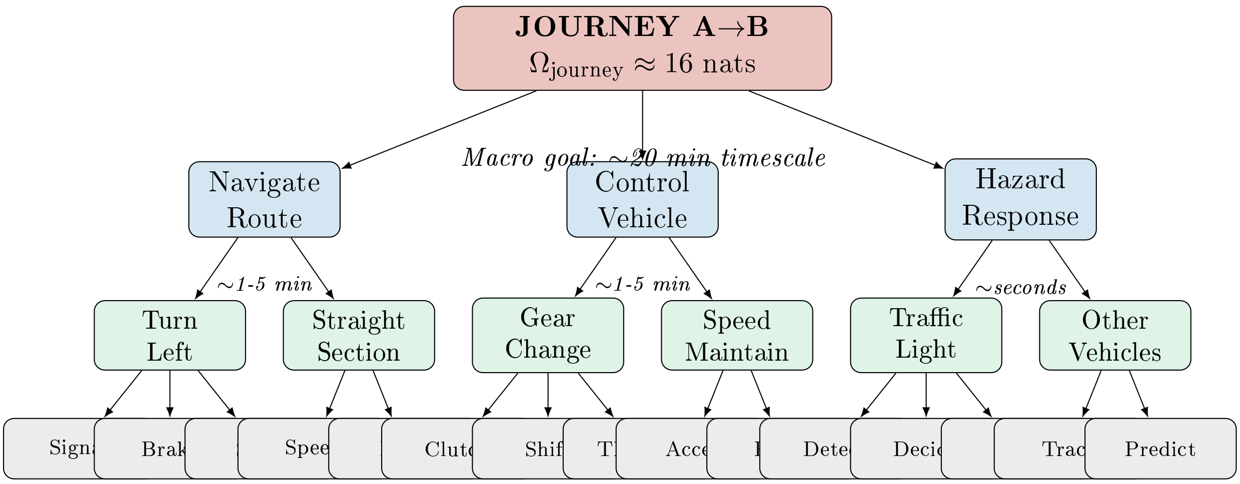

Figure 1: Hierarchical goal structure of a driving journey. Each level contains nested CRR cycles, all governed by the same $\Omega \approx 16$ nats threshold for "inside-outside match."

4 Detailed Analysis: Micro-Goals in FEP and CRR

4.1 Example 1: Gear Change

Consider shifting from 2nd to 3rd gear as RPM approaches 3000.

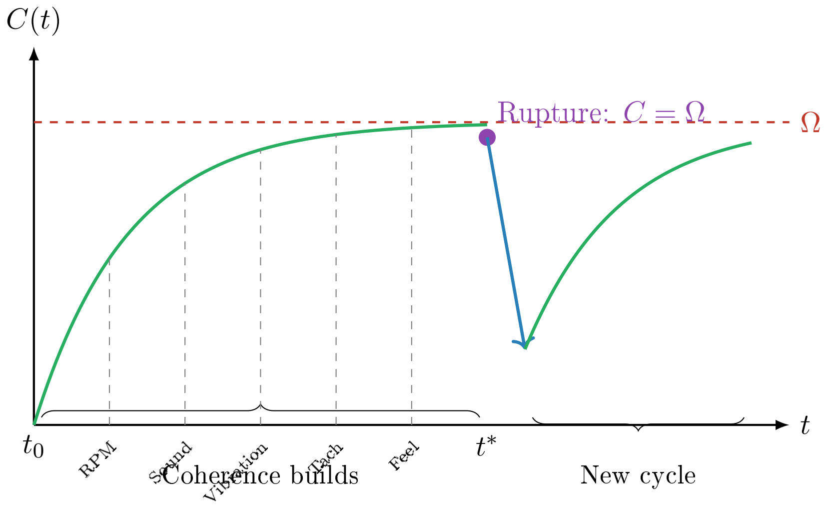

Figure 2: CRR dynamics for a gear change. Coherence accumulates from multiple sensory channels until rupture triggers the shift action.

FEP Formulation 4.1: Gear Change in FEP Terms

Generative model: $p(o, s) = p(\text{RPM}, \text{sound}, \text{vibration}|\text{gear}, \text{speed}) \cdot p(\text{gear}, \text{speed})$
Goal preference: $p(o|\mathcal{G}) = \mathcal{N}(\text{RPM}; 2000, \sigma^2)$ (prefer RPM around 2000)
Free energy before shift (RPM at 3000):

$$F_{\text{before}} = \frac{1}{2\sigma^2}(3000 - 2000)^2 + \text{const} = \frac{10^6}{2\sigma^2} \tag{9}$$

Expected free energy of shift policy π_{shift}:

$$G(\pi_{\text{shift}}) = \mathbb{E}\left[\frac{1}{2\sigma^2}(\text{RPM}_{\text{after}} - 2000)^2\right] \approx \frac{(2100 - 2000)^2}{2\sigma^2} = \frac{10^4}{2\sigma^2} \tag{10}$$

Action selection: $\pi^* = \arg\min_\pi G(\pi) = \pi_{\text{shift}}$ (shift minimizes expected free energy)

CRR Formulation 4.1: Gear Change in CRR Terms

Coherence accumulation:

$$C(t) = \int_0^t [L_{\text{RPM}}(\tau) + L_{\text{sound}}(\tau) + L_{\text{vibration}}(\tau) + L_{\text{tach}}(\tau)]\, d\tau \tag{11}$$

Each channel contributes $\sim 4-6$ bits to coherence:

- L_{RPM}: Tachometer reading $\rightarrow$ 5 bits
- L_{sound}: Engine pitch $\rightarrow$ 4 bits
- $L_{\text{vibration}}$: Proprioceptive $\rightarrow$ 5 bits
- L_{feel}: Acceleration feel $\rightarrow$ 4 bits

Total at rupture: $C(t^*) = 18-20$ bits $\approx 13-14$ nats $\rightarrow \Omega$
Rupture: "Now is the moment to shift" – inside (prediction of optimal shift point) matches outside (sensory evidence confirms)
Regeneration:

$$R[\phi_{\text{motor}}] = \int \phi_{\text{motor}}(\tau) \cdot e \cdot \Theta(t-\tau)\, d\tau \tag{12}$$

Historical motor pattern (clutch-shift-throttle sequence) is weighted by e and executed.

Inside-Outside Match 4.1: The Inside-Outside Match for Gear Change

Inside (prediction/goal): "When RPM $\approx$ 3000, shift to maintain efficiency"
Outside (observation): Tachometer, engine sound, vibration all indicate RPM $\approx$ 3000
Match condition:

$$\underbrace{C(t^*)}_{\substack{\text{Accumulated}\\ \text{evidence}}} = \underbrace{\Omega}_{\substack{\text{Goal}\\ \text{precision}}} \quad \Leftrightarrow \quad \underbrace{F \rightarrow F_{\text{min}}}_{\substack{\text{Free energy}\\ \text{minimized}}} \tag{13}$$

Outcome branches:

- **Success** (smooth shift): Regeneration *reinforces* motor program $\rightarrow$ lower Ω next time (more automatic)
- **Failure** (grind): Regeneration *rewires* timing $\rightarrow$ adjust clutch-throttle coordination

4.2 Example 2: Traffic Light Response

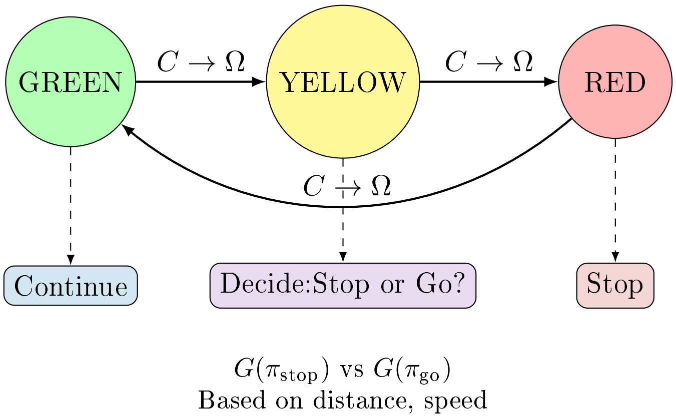

Figure 3: Traffic light state transitions and corresponding agent decisions. Each state transition represents a CRR cycle with rupture triggering action selection.

FEP Formulation 4.2: Traffic Light in FEP: The Yellow Light Dilemma

At yellow light onset, the agent must select between policies:
Policy 1 (π_{stop}): Brake and stop before intersection

$$G(\pi_{\text{stop}}) = \underbrace{0}_{\text{epistemic}} + \underbrace{D_{KL}[q(o|\pi_{\text{stop}}) \| p(o|\mathcal{G}_{\text{safety}})]}_{\text{pragmatic: safety cost}} \tag{14}$$

Policy 2 (π_{go}): Accelerate through intersection

$$G(\pi_{\text{go}}) = \underbrace{H[p(o|\pi_{\text{go}})]}_{\text{epistemic: uncertainty}} + \underbrace{D_{KL}[q(o|\pi_{\text{go}}) \| p(o|\mathcal{G}_{\text{safety}})]}_{\text{pragmatic: risk if red}} \tag{15}$$

Decision boundary: At distance d^* and speed v^*:

$$G(\pi_{\text{stop}}) = G(\pi_{\text{go}}) \quad \Rightarrow \quad \text{“Point of no return”} \tag{16}$$

CRR Formulation 4.2: Traffic Light in CRR: Evidence Accumulation

Coherence channels:

$$L_{\text{visual}}(t) = \text{Light color recognition} \approx 3 \text{ bits} \tag{17}$$
$$L_{\text{distance}}(t) = \text{Distance to intersection} \approx 5 \text{ bits} \tag{18}$$
$$L_{\text{speed}}(t) = \text{Current velocity} \approx 4 \text{ bits} \tag{19}$$
$$L_{\text{traffic}}(t) = \text{Other vehicles} \approx 4 \text{ bits} \tag{20}$$
$$L_{\text{timing}}(t) = \text{Yellow duration estimate} \approx 3 \text{ bits} \tag{21}$$

Total coherence: $C = 19 - 20$ bits ≈ 14 nats
Rupture condition: When $C = \Omega \approx 16$ nats, decision crystallizes:

$$\text{Action} = \begin{cases} \text{Brake} & \text{if } C_{\text{stop}} > C_{\text{go}} \\ \text{Accelerate} & \text{if } C_{\text{go}} > C_{\text{stop}} \end{cases} \tag{22}$$

4.3 Example 3: Lane Change Maneuver

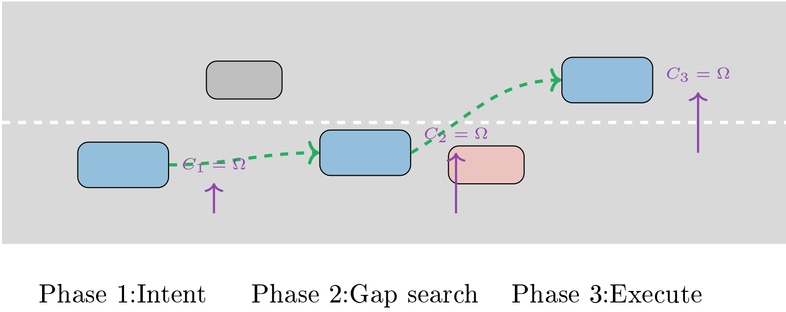

Figure 4: Lane change as three nested CRR cycles: (1) form intent, (2) identify gap, (3) execute maneuver. Each phase has its own coherence-rupture-regeneration cycle.

FEP Formulation 4.3: Lane Change: Multi-Phase Active Inference

Phase 1 - Intent Formation:

$$G(\pi_{\text{change}}) = \mathbb{E}\left[\text{time saved}\right] - \lambda \cdot \mathbb{E}\left[\text{risk}\right] \tag{23}$$

Phase 2 - Gap Assessment: Generative model predicts other vehicle trajectories:

$$p(x_{\text{other}}(t + \Delta t) | x_{\text{other}}(t), v_{\text{other}}) = \mathcal{N}(x + v\Delta t, \sigma^2_{\text{pred}}) \tag{24}$$

Gap sufficiency: Gap > Vehicle length + 2 · Safety margin
Phase 3 - Execution: Motor commands minimize:

$$F_{\text{motor}} = \|x_{\text{actual}}(t) - x_{\text{planned}}(t)\|^2_{\Pi} \tag{25}$$

where Π is precision (confidence in motor plan).

CRR Formulation 4.3: Lane Change: Nested CRR Cycles

Cycle 1 - Intent ($\Omega_1 \approx 16$ nats):

- Coherence: Traffic density, speed differential, navigation need
- Rupture: "Yes, I will change lanes"
- Regeneration: Activate gap-search mode

Cycle 2 - Gap ($\Omega_2 \approx 16$ nats):

- Coherence: Mirror checks, blind spot, closing speeds
- Rupture: "Gap is sufficient"
- Regeneration: Initiate steering + signal

Cycle 3 - Execute ($\Omega_3 \approx 16$ nats):

- Coherence: Steering angle, lane position, other vehicle response
- Rupture: "Maneuver complete"
- Regeneration: Return to lane-keeping mode

Note: Same $\Omega \approx 16$ nats at each level, but different coherence channels.

5 The Complete Journey: A to B

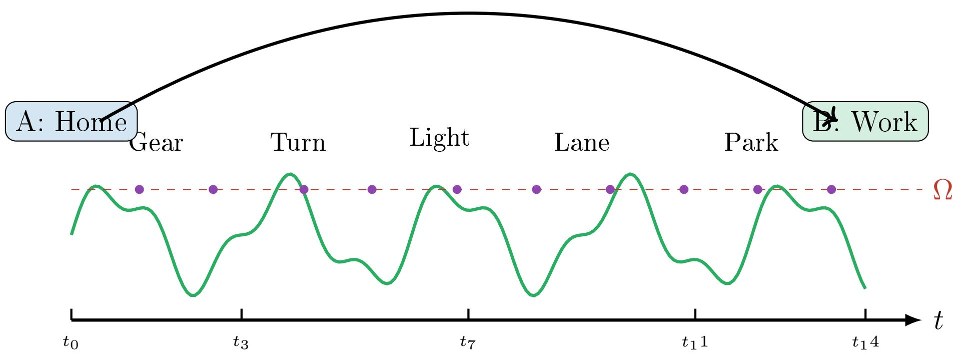

Figure 5: The complete journey from A to B shown as oscillating coherence punctuated by rupture events. Each peak crossing Ω represents a micro-goal completion (gear change, turn, traffic response, etc.).

5.1 Mathematical Summary of Journey

FEP Formulation 5.1: Journey in FEP: Hierarchical Policy Selection

The complete journey is a hierarchical policy:

$$\pi_{\text{journey}} = \{\pi_{\text{nav}}, \pi_{\text{vehicle}}, \pi_{\text{hazard}}\} \tag{26}$$

Total expected free energy:

$$G(\pi_{\text{journey}}) = \sum_{k=1}^{N} \gamma^k \cdot G(\pi_k) \tag{27}$$

where $\gamma < 1$ is temporal discounting and N is number of sub-goals.
Success criterion: Arrive at B with:

$$D_{KL}[q(\text{location}) \| \delta_B] < \epsilon \tag{28}$$

(Belief about location concentrated at B)

CRR Formulation 5.1: Journey in CRR: Nested Cycle Cascade

The journey consists of N nested CRR cycles:

$$\text{Journey} = \bigcup_{k=1}^{N} \text{CRR}_k = \bigcup_{k=1}^{N} (C_k \to \Omega \to R_k) \tag{29}$$

Total information processed:

$$I_{\text{total}} = \sum_{k=1}^{N} C_k(t_k^*) \approx N \times 16 \text{ nats} \tag{30}$$

For a 20-minute journey with $\sim$100 micro-goals:

$$I_{\text{total}} \approx 100 \times 16 = 1600 \text{ nats} \approx 2300 \text{ bits} \tag{31}$$

Rate: $\frac{2300 \text{ bits}}{1200 \text{ s}} \approx 2$ bits/s conscious processing (matches cognitive bandwidth estimates!)

6 The Unified Picture

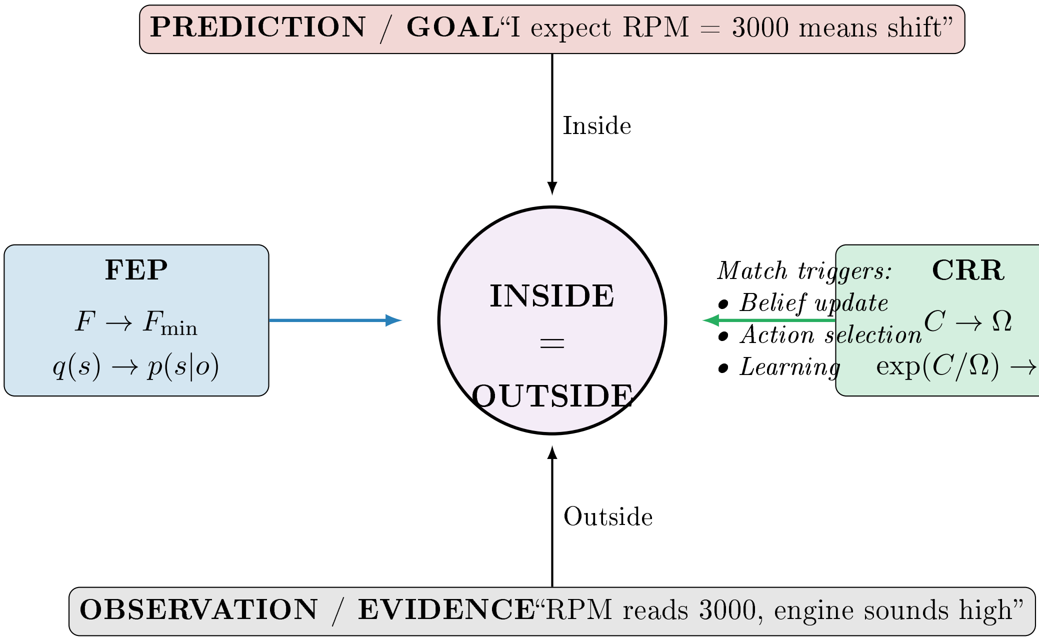

Figure 6: The unified picture: Both FEP ($F \to F_{\min}$) and CRR ($C \to \Omega$) describe the same phenomenon—the moment when internal predictions align with external observations, triggering state transitions.

6.1 Why 16 Nats is the Universal Threshold

Inside-Outside Match 6.1: The Information-Theoretic Interpretation

$\Omega \approx 16$ nats ≈ 23 bits represents:

1. **Goal specification precision**: To specify a goal ("shift gear", "stop at light") requires $\sim$23 bits to distinguish from $2^{23} \approx 8$ million alternatives.
2. **Evidence sufficiency**: To confirm a goal is achieved requires accumulating $\sim$23 bits of sensory evidence across multiple channels.
3. **Error correction capacity**: Biological systems can reliably correct errors up to $\sim 2^{23}$ states before requiring reorganization.
4. **Thermodynamic bound**: At body temperature, $16 \times k_B T \approx 40$ kJ/mol—the typical activation barrier for molecular conformational changes.

The convergence of cognitive, biological, and thermodynamic constraints on this value suggests it is a **fundamental constant of adaptive systems**.

7 Outcome-Dependent Regeneration

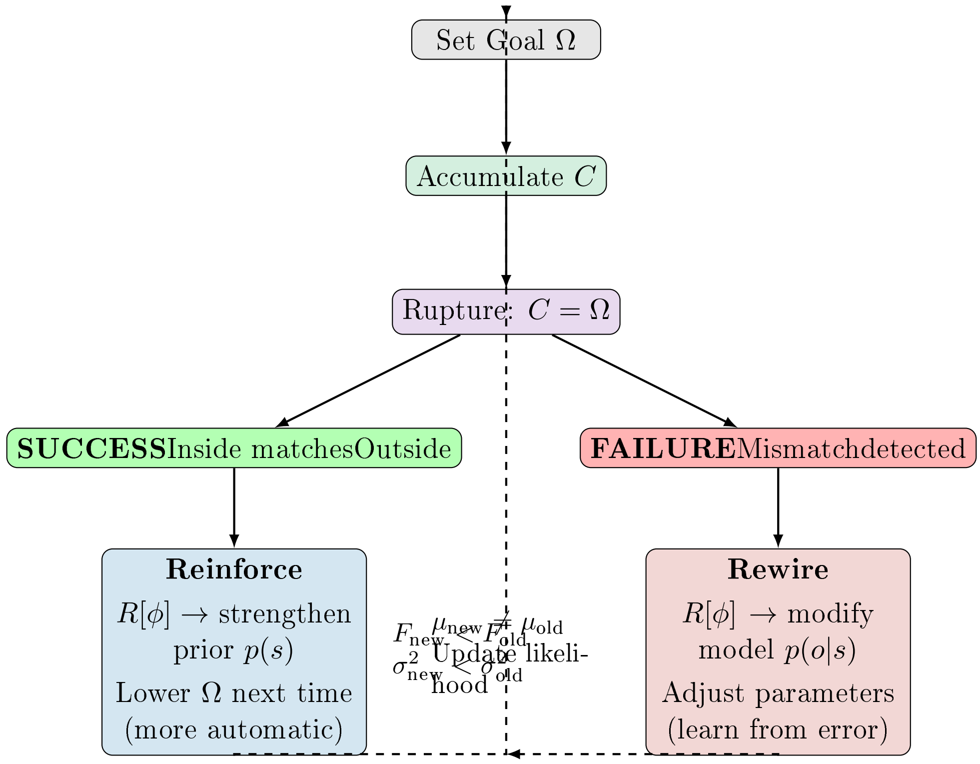

Figure 7: The bifurcation at rupture: success leads to reinforcement (tighter priors), failure leads to rewiring (updated likelihood). Both paths use the regeneration operator $R[\phi]$ but with different effects on the generative model.

FEP Formulation 7.1: Success: Precision Increase

When prediction matches observation:

$$\Pi_{\text{new}} = \Pi_{\text{old}} + \alpha \cdot \Pi_{\text{old}} = (1 + \alpha)\Pi_{\text{old}} \tag{32}$$

where Π is precision (inverse variance) and $\alpha > 0$ is learning rate.
Effect: The same action becomes more automatic, requiring less conscious monitoring.

CRR Formulation 7.1: Success: Lower Effective Ω

Successful pattern completion reduces the threshold for future recognition:

$$\Omega_{\text{new}} = \Omega_{\text{old}} - \beta \cdot \ln(1 + n_{\text{success}}) \tag{33}$$

where n_{success} counts successful completions.
Effect: Expert drivers shift gears with minimal coherence accumulation—the pattern is automatized.

FEP Formulation 7.2: Failure: Likelihood Update

When prediction fails:

$$p_{\text{new}}(o|s) \propto p_{\text{old}}(o|s) \cdot \exp\left(-\frac{\varepsilon^2}{2\sigma^2}\right) \tag{34}$$

where $\varepsilon = o_{\text{actual}} - o_{\text{predicted}}$ is prediction error.
Effect: The model of how states generate observations is updated to reduce future errors.

CRR Formulation 7.2: Failure: Regeneration as Contrast

The regeneration operator weights historical states, but now as *what to change*:

$$R_{\text{contrast}}[\phi] = \int \phi(\tau) \cdot e \cdot (1 - \text{similarity}(\tau, t^*)) \cdot \Theta(t - \tau)\, d\tau \tag{35}$$

States most similar to the failed attempt get downweighted; dissimilar states (potential alternatives) get upweighted.

8 Conclusion: CRR as the Grammar of Active Inference

We have demonstrated that the CRR framework provides the **dynamical grammar** underlying active inference:

1. **Coherence** $C(t)$ corresponds to accumulated evidence / negative free energy
2. **Threshold** Ω corresponds to goal precision / prior certainty
3. **Rupture** $C = \Omega$ corresponds to the inside-outside match / free energy minimum
4. **Regeneration** $R[\phi]$ corresponds to posterior update / policy selection

The universal threshold $\Omega \approx 16$ nats represents the information capacity required for:

- Specifying a goal with sufficient precision
- Accumulating evidence sufficient for confident action
- Maintaining error-correction in biological computation

A driving journey from A to B consists of ∼100 nested CRR cycles, each following the same grammar: accumulate coherence toward a goal, rupture when evidence matches expectation, regenerate to either reinforce success or rewire after failure. This elegant recursive structure scales from millisecond motor adjustments to hour-long navigation plans, always with the same fundamental threshold of ∼16 nats marking the moment when inside meets outside.

The Central Equation

$$\underbrace{C(t^*) = \Omega}_{\text{CRR: Rupture}} \iff \underbrace{F(q^*) = F_{\min}}_{\text{FEP: Equilibrium}} \iff \underbrace{\text{Inside} = \text{Outside}}_{\text{Phenomenology: Recognition}}$$