

# FEP-CRR Correspondence: A Working Exploration

Observed Mathematical Patterns Between  
Free Energy Principle and Coherence-Rupture-Regeneration

Research Notes — November 2025

**Note:** This document presents observed correspondences between FEP and CRR frameworks. These are working conjectures based on mathematical patterns, simulation behavior, and cross-domain analysis. Consider this a collaborative exploration tool rather than established proof.

## Contents

# 1 Core Correspondences

## 1.1 State Variables

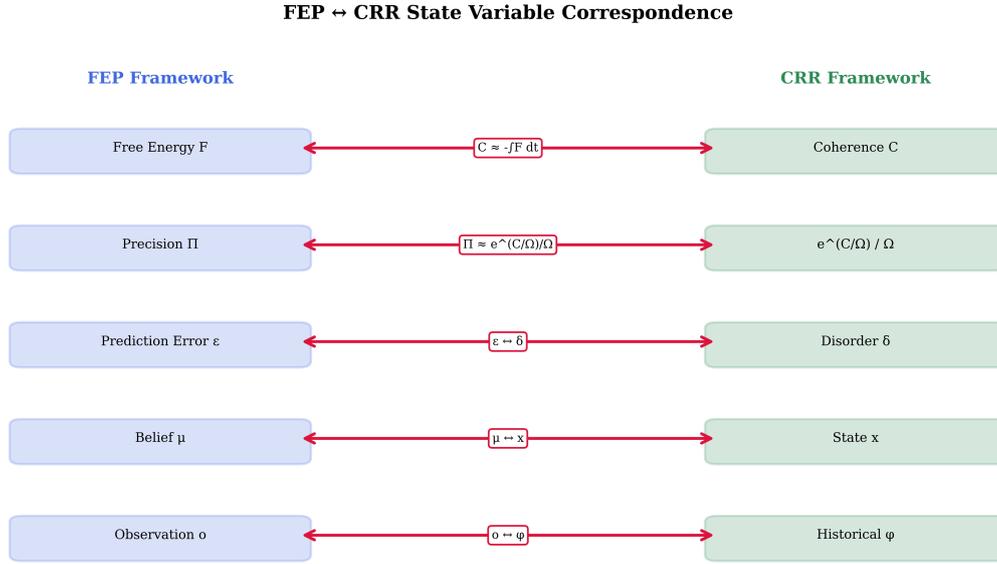


Figure 1: FEP and CRR state variable mappings

### State Variable Mapping

$$\mathbf{FEP:} \text{ Free Energy } F(t) \leftrightarrow \text{ Coherence } C(t) \quad \mathbf{:CRR} \quad (1)$$

$$\mathbf{FEP:} \text{ Surprise } -\ln P(o|\mu) \leftrightarrow \text{ Memory Density } -L(x, t) \quad \mathbf{:CRR} \quad (2)$$

$$\mathbf{FEP:} \text{ Precision } \Pi(t) \leftrightarrow \frac{1}{\Omega} \cdot e^{C(t)/\Omega} \quad \mathbf{:CRR} \quad (3)$$

$$\mathbf{FEP:} \text{ Prediction Error } \varepsilon(t) \leftrightarrow \text{ Disorder } \delta_{\text{local}}(t) \quad \mathbf{:CRR} \quad (4)$$

## 1.2 Fundamental Relation: Coherence as Integrated Free Energy Reduction

### FEP: Variational Free Energy

$$F = \mathbb{E}_{Q(\mu)}[\ln Q(\mu) - \ln P(o, \mu)] = D_{KL}[Q(\mu) || P(\mu|o)] - \ln P(o) \quad (5)$$

Where:

- $Q(\mu)$ : Recognition density (internal beliefs)
- $P(o, \mu)$ : Joint probability of observations and states
- $D_{KL}$ : Kullback-Leibler divergence

## CRR: Coherence Accumulation

$$C(x, t) = \int_0^t L(x, \tau) d\tau \quad \text{where} \quad L(x, \tau) = \frac{dC}{d\tau} \quad (6)$$

Where:

- $C(x, t)$ : Accumulated coherence (integrated memory)
- $L(x, \tau)$ : Memory density (rate of coherence change)

## Observed Correspondence

$$L(x, t) = -\frac{dF}{dt} \quad \Rightarrow \quad C(x, t) = F(x, 0) - F(x, t) + C_0 \quad (7)$$

**Interpretation:** Coherence accumulation rate equals free energy reduction rate. Systems that minimize surprise (FEP) accumulate memory (CRR).

## 2 Precision-Temperature Correspondence

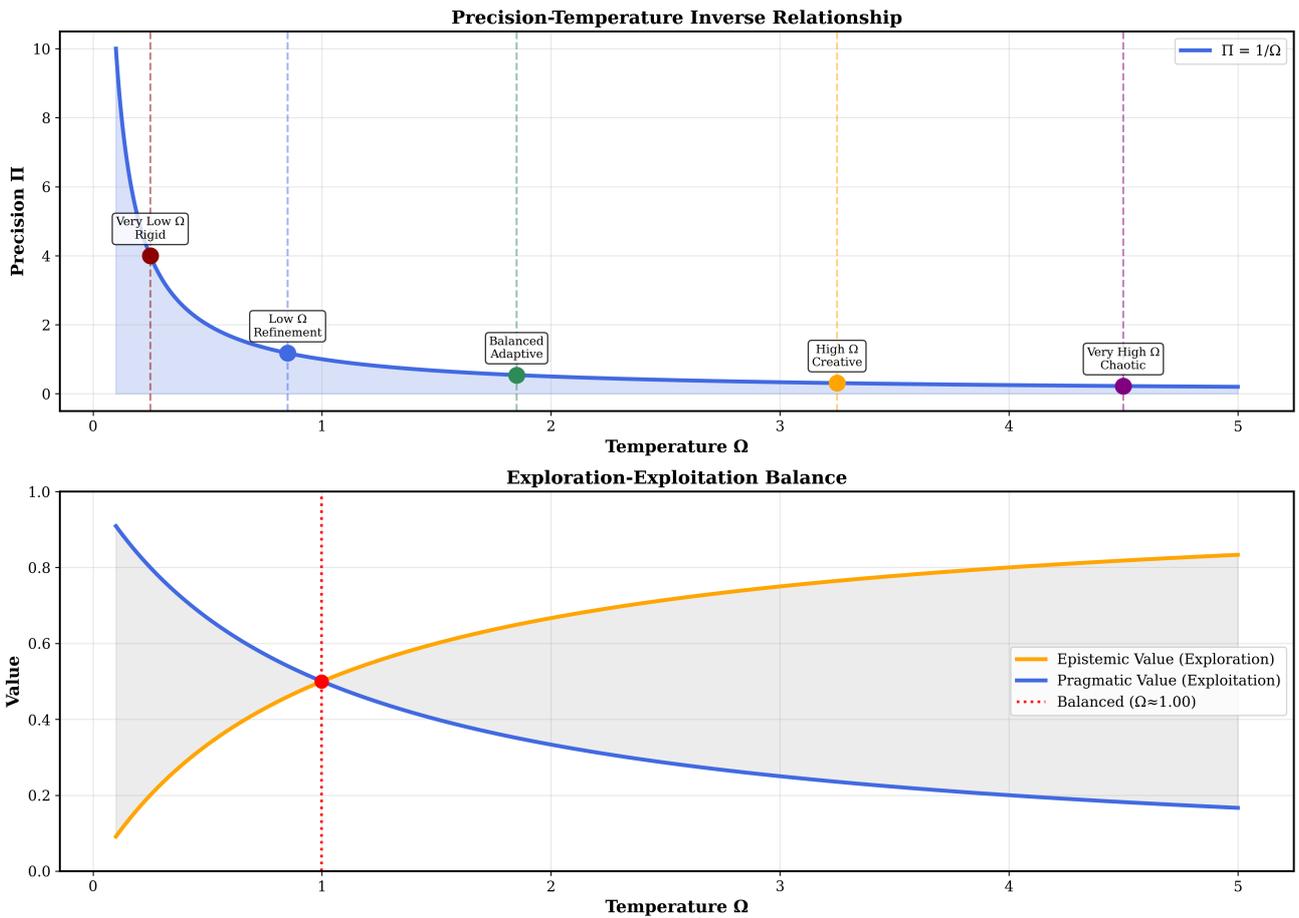


Figure 2: The Omega-Precision relationship across exploration-exploitation spectrum

## 2.1 Precision in FEP

**Precision**  $\Pi$  quantifies confidence in predictions:

$$\Pi = \frac{1}{\sigma^2} = (\text{inverse variance of prediction errors}) \quad (8)$$

Belief updates are precision-weighted:

$$\frac{d\mu}{dt} = -\frac{\partial F}{\partial \mu} = \Pi \cdot \varepsilon \quad (9)$$

Where  $\varepsilon = o - g(\mu)$  is prediction error and  $g(\mu)$  is the generative prediction.

## 2.2 Temperature (Omega) in CRR

**Temperature**  $\Omega$  controls memory weighting:

$$R[\chi](x, t) = \int_0^t \phi(x, \tau) \cdot e^{C(x, \tau)/\Omega} \cdot K(t - \tau) d\tau \quad (10)$$

Future states regenerate from exponentially-weighted past:

$$\text{Weight}(\tau) = \exp\left(\frac{C(\tau)}{\Omega}\right) \quad (11)$$

## 2.3 Observed Equivalence

Precision

$$\Pi(t) \approx \frac{e^{C(t)/\Omega}}{\Omega} \quad \text{or equivalently} \quad \Omega \approx \frac{C(t)}{\ln(\Pi(t) \cdot \Omega_{\text{ref}})} \quad (12)$$

**Rationale:**

$$\text{High } C \text{ (learned)} \Rightarrow \text{High precision (confident)} \quad (13)$$

$$\text{High } \Omega \text{ (hot)} \Rightarrow \text{Low effective precision (uncertain)} \quad (14)$$

$$\Pi \propto \exp(C/\Omega) \quad (\text{exponential growth with learning}) \quad (15)$$

# 3 Expected Free Energy and Action Selection

## 3.1 Expected Free Energy Decomposition

FEP: Expected Free Energy

Policy  $\pi$  selected to minimize expected future surprise:

$$G(\pi) = \underbrace{\mathbb{E}_Q[\ln Q(s_\tau|\pi) - \ln Q(s_\tau|o_\tau, \pi)]}_{\text{Epistemic Value (Information Gain)}} - \underbrace{\mathbb{E}_Q[\ln P(o_\tau|C)]}_{\text{Pragmatic Value (Goal Achievement)}} \quad (16)$$

Simplifies to:

$$G(\pi) = -I[s_\tau; o_\tau|\pi] + \mathbb{E}_Q[-\ln P(o_\tau|C)] \quad (17)$$

$$= \text{Ambiguity} - \text{Expected Utility} \quad (18)$$

## CRR: Expected Coherence Gain

Action selection maximizes expected coherence accumulation:

$$\pi^* = \arg \max_{\pi} \mathbb{E} \left[ \int_t^T L(x(\tau), \pi) d\tau \right] = \arg \max_{\pi} \mathbb{E}[\Delta C(\pi)] \quad (19)$$

Decompose by exploration vs. exploitation:

$$\mathbb{E}[\Delta C(\pi)] = \underbrace{\int_t^T I[x(\tau); \text{past}|\pi] \cdot e^{C(\tau)/\Omega} d\tau}_{\text{Epistemic Coherence Gain}} + \underbrace{\int_t^T \ln P(\text{preferred}|\pi) d\tau}_{\text{Pragmatic Coherence Gain}} \quad (20)$$

## Action Selection Mapping

Minimize $G(\pi)$	$\leftrightarrow$	Maximize $\mathbb{E}[\Delta C(\pi)]$	(21)
Epistemic value	$\leftrightarrow$	Exploration (high $\Omega$ )	
Pragmatic value	$\leftrightarrow$	Exploitation (low $\Omega$ )	

**Effective Policy Weighting:**

$$\text{Score}(\pi) = \Omega \cdot \text{Epistemic Gain}(\pi) + \frac{1}{\Omega} \cdot \text{Pragmatic Gain}(\pi) \quad (22)$$

## 4 Exploration-Exploitation Spectrum

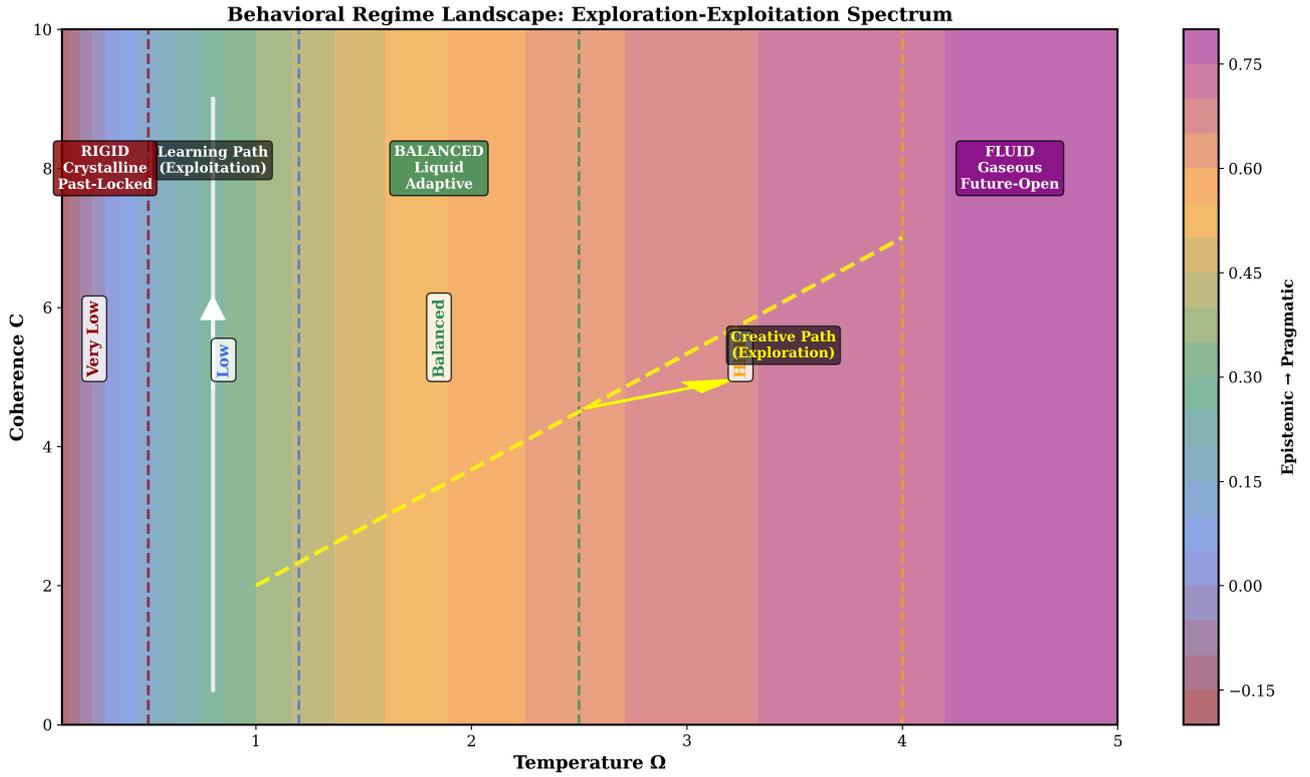


Figure 3: Behavioral regimes across Omega/Precision parameter space

Table 1: Omega Regimes: Observed Phenomenology from Simulations

$\Omega$ Range	$\Pi$ Range	Epistemic	Pragmatic	Behavior	Metaphor
$< 0.5$	$> 4.0$	0.3	0.9	Rigid	Crystalline
$0.5 - 1.2$	$2.0 - 4.0$	0.5	0.7	Refinement	Structured
$1.2 - 2.5$	$0.8 - 2.0$	0.6-0.7	0.5	Balanced	Liquid
$2.5 - 4.0$	$0.4 - 0.8$	0.7-0.8	0.3	Creative	Fluid
$> 4.0$	$< 0.4$	$> 0.8$	$< 0.2$	Chaotic	Gaseous

### Observed Pattern:

- Low  $\Omega$  (high precision): Past dominates  $\rightarrow$  Exploitation  $\rightarrow$  "Memory chains me to certainty"
- High  $\Omega$  (low precision): Future open  $\rightarrow$  Exploration  $\rightarrow$  "Formless potential, structure dissolves"
- Balanced  $\Omega$ : Adaptive switching  $\rightarrow$  "Experience guides, curiosity propels"

## 5 Rupture as Model Switching

### 5.1 Model Inadequacy in FEP

Model  $m$  becomes inadequate when variational bound loses tightness:

$$D_{KL}[Q(\mu|m)||P(\mu|o, m)] > D_{\text{crit}} \quad (23)$$

Triggers model selection:

$$m^* = \arg \max_m \ln P(o|m) \approx \arg \min_m F(m) \quad (24)$$

**Signature:** Gradient explosion in free energy landscape:

$$\|\nabla_{\mu} F\|^2 \rightarrow \infty \quad (\text{no minimum exists under current model}) \quad (25)$$

### 5.2 Rupture in CRR

Rupture event as Dirac delta discontinuity:

$$\delta(t - t_0) \quad \text{where} \quad \int_{t_0^-}^{t_0^+} \delta(t - t_0) dt = 1 \quad (26)$$

Triggered when:

$$C(x, t) \geq C_{\text{crit}} \quad \text{or} \quad \frac{\partial^2 C}{\partial t^2} > \text{threshold (acceleration)} \quad (27)$$

Post-rupture coherence reset:

$$C(t_0^+) = \alpha \cdot C(t_0^-) \quad \text{where} \quad \alpha \in [0, 1] \quad (28)$$

### Rupture Correspondence

$$\begin{array}{l}
 \text{Rupture Condition: } F(m_{\text{current}}) > F_{\text{threshold}} \\
 \updownarrow \\
 C(t) \geq C_{\text{crit}} \\
 \\
 \text{Model Switch: } m \rightarrow m' \\
 \updownarrow \\
 \text{Coherence Reset: } C \rightarrow \alpha C \\
 \\
 \text{Mechanism: Bayesian Model Selection} \\
 \updownarrow \\
 \text{History-Weighted Regeneration}
 \end{array} \quad (29)$$

## 6 Regeneration as Bayesian Model Averaging

### 6.1 Bayesian Model Evidence

Evidence for model  $m$  given data  $\mathcal{D}$ :

$$P(m|\mathcal{D}) \propto P(\mathcal{D}|m) \cdot P(m) \quad (30)$$

Since  $P(\mathcal{D}|m) = \exp(-F_m)$ :

$$P(m|\mathcal{D}) \propto \exp(-F_m) \quad (31)$$

Model averaging:

$$\mathbb{E}[x] = \sum_m P(m|\mathcal{D}) \cdot x_m \quad (32)$$

### 6.2 CRR Regeneration Operator

Future state reconstructed from weighted historical states:

$$R[\chi](x, t) = \int_0^t \phi(x, \tau) \cdot \underbrace{e^{C(x, \tau)/\Omega}}_{\text{Evidence Weight}} \cdot K(t - \tau) d\tau \quad (33)$$

Where:

- $\phi(x, \tau)$ : Historical field (past states)
- $e^{C(\tau)/\Omega}$ : Exponential coherence weighting ( model evidence)
- $K(t - \tau)$ : Memory kernel (temporal decay)

#### Regeneration Mapping

$$e^{C_m(\tau)/\Omega} \leftrightarrow P(\mathcal{D}|m) = e^{-F_m} \quad (34)$$

**Interpretation:**

$$\text{High } C_m \Rightarrow \text{High evidence weight} \quad (35)$$

$$\text{Low } F_m \Rightarrow \text{High model probability} \quad (36)$$

$$R[\chi] = \text{Weighted average over historical models} \quad (37)$$

**Implication:** Regeneration implements Bayesian model averaging where "models" are past system states weighted by their prediction accuracy (coherence).

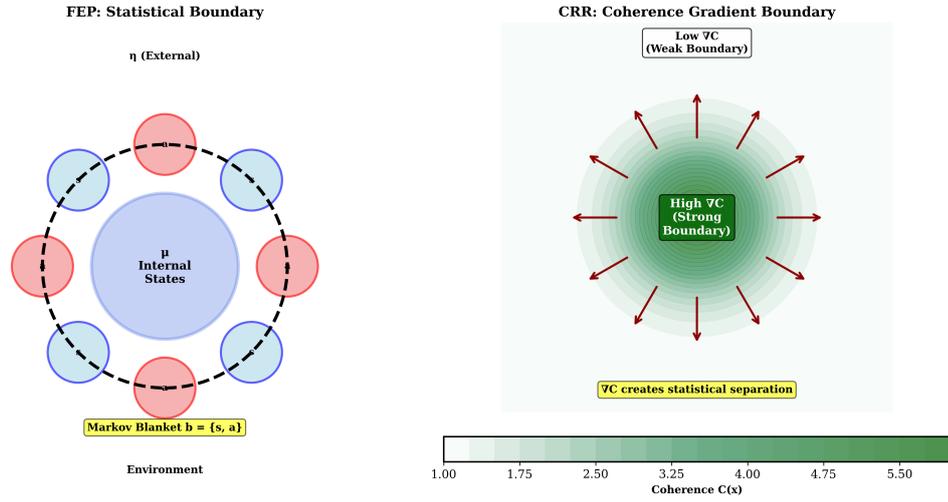


Figure 4: Markov blanket as coherence boundary

## 7 Markov Blankets and Boundaries

### 7.1 Markov Blanket in FEP

Statistical boundary rendering internal ( $\mu$ ) and external ( $\eta$ ) states conditionally independent:

$$P(\mu, \eta | s, a) = P(\mu | s, a) \cdot P(\eta | s, a) \quad (38)$$

Where:

- $s$ : Sensory states (affected by external, affect internal)
- $a$ : Active states (affected by internal, affect external)
- Blanket  $b = \{s, a\}$  mediates all coupling

**Existence Condition:** Sparse coupling in dynamical system maintains conditional independence.

### 7.2 Coherence Field as Boundary

Coherence field  $C(x, t)$  defines statistical separation:

$$\text{Gradient } \nabla C \text{ creates "memory terrain"} \quad (39)$$

High  $|\nabla C|$  regions = strong boundaries (system maintains identity)

Low  $|\nabla C|$  regions = weak boundaries (system merges with environment)

**Boundary Maintenance:**

$$\frac{\partial C}{\partial t} = L(x, t) = \text{rate of boundary reinforcement} \quad (40)$$

## Boundary Correspondence

Markov Blanket $b$	$\leftrightarrow$	High- $\nabla C$ Surface	(41)
Blanket Maintenance	$\leftrightarrow$	$L(x, t) > 0$	
Blanket Dissolution	$\leftrightarrow$	$L(x, t) < 0$	

**Insight:** Markov blankets are not static—they require active maintenance. CRR’s coherence accumulation ( $L > 0$ ) corresponds to FEP’s free energy minimization maintaining the blanket.

## 8 Non-Markovian Memory Structure

### 8.1 Markovian vs. Non-Markovian Dynamics

#### Standard FEP (Markovian)

Present state sufficient for prediction:

$$P(x_{t+1}|x_t, x_{t-1}, \dots, x_0) = P(x_{t+1}|x_t) \quad (42)$$

Belief update depends only on current state:

$$\frac{d\mu}{dt} = f(\mu_t, o_t) \quad (43)$$

#### CRR (Non-Markovian)

Entire history matters through coherence integral:

$$C(t) = \int_0^t L(\tau) d\tau \quad \Rightarrow \quad \text{Future depends on full past} \quad (44)$$

Regeneration creates path-dependence:

$$R(t) = \int_0^t \phi(\tau) \cdot e^{C(\tau)/\Omega} K(t - \tau) d\tau \quad (45)$$

**Observed Pattern:** Systems transition from Markovian  $\rightarrow$  Non-Markovian as coherence accumulates:

$$\text{Effective Memory Depth} = \int_0^t e^{C(\tau)/\Omega} d\tau \quad (46)$$

- $C \approx 0$ : Depth  $\approx t$  (memoryless, Markovian)
- $C \gg \Omega$ : Depth  $\propto e^{C/\Omega}$  (deep memory, non-Markovian)

## Hierarchical CRR: Multi-Scale Coherence

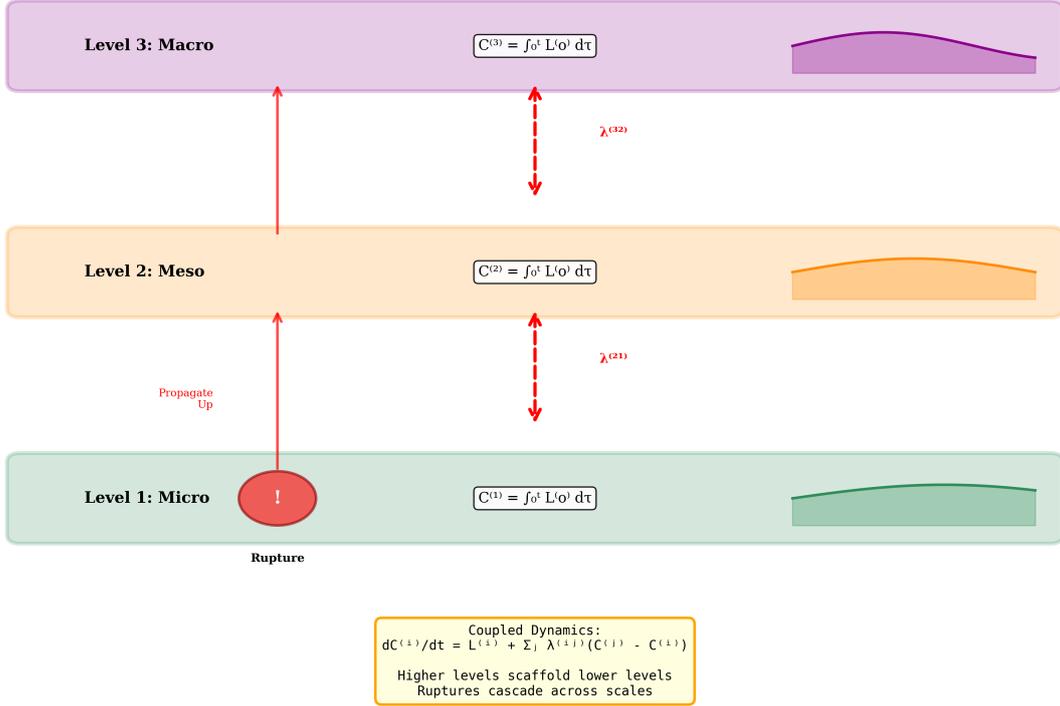


Figure 5: Multi-scale CRR with cross-level coherence coupling

## 9 Hierarchical Structure

### 9.1 Hierarchical Active Inference

Multi-level generative model with hierarchical prediction errors:

$$\text{Level } i : F^{(i)} = \mathbb{E}[\ln Q(\mu^{(i)}) - \ln P(s^{(i-1)}, \mu^{(i)} | \mu^{(i+1)})] \quad (47)$$

$$\text{Error}^{(i)} : \varepsilon^{(i)} = s^{(i-1)} - g^{(i)}(\mu^{(i)}) \quad (48)$$

$$\text{Update}^{(i)} : \dot{\mu}^{(i)} = \Pi^{(i)} \varepsilon^{(i)} + \Pi^{(i+1)} \varepsilon^{(i+1)} \quad (49)$$

## 9.2 Multi-Scale CRR

Coherence at each scale with cross-scale coupling:

$$C^{(i)}(t) = \int_0^t \left[ L^{(i)}(\tau) + \sum_{j \neq i} \lambda^{(ij)} (C^{(j)}(\tau) - C^{(i)}(\tau)) \right] d\tau \quad (50)$$

Where:

- $L^{(i)}$ : Local coherence accumulation at scale  $i$
- $\lambda^{(ij)}$ : Coupling strength between scales
- Cross-scale term: Higher levels scaffold lower levels

**Rupture Propagation:**

$$\text{If } C^{(i)} > C_{\text{crit}}^{(i)} \Rightarrow \begin{cases} \text{Propagate up if } \Delta F^{(i)} > \Delta F^{(i+1)} \\ \text{Propagate down via top-down predictions} \end{cases} \quad (51)$$

## 10 Parameter Identification Methodology

### 10.1 Extracting $\Omega$ from System Observations

General Methodology (from original document)

To determine  $\Omega$  for a given system:

#### 1. Identify Memory Density $L(x, t)$ :

- What increases system integration/memory?
- What decreases coherence/causes forgetting?

**Examples Across Domains:**

$$\text{Ocean waves: } L = \frac{v_{\text{orbital}}}{10} \cdot \frac{E_{\text{wave}}}{1000} \quad (52)$$

$$\text{Hurricanes: } L = \left( 0.2 + 0.8 \frac{v_{\text{wind}}}{220} \right) \times f_{\text{env}} \quad (53)$$

$$\text{Bees: } L = r_{\text{nectar}} + a_{\text{learning}} - f_{\text{fatigue}} \quad (54)$$

$$\text{Neural: } L = \text{synaptic integration} + \text{attention} - \text{fatigue} \quad (55)$$

## FEP-Guided Parameter Extraction

### Step-by-Step Process:

1. **Measure Time Series:**  $\{o(t), \mu(t)\}$  (observations and inferred states)
2. **Fit Generative Model:**  $P(o|\mu, \theta)$  using maximum likelihood
3. **Compute Free Energy:**

$$F(t) = -\ln P(o(t)|\mu(t), \theta) + H[Q(\mu)] \quad (56)$$

4. **Estimate Coherence:**

$$C(t) \approx F(0) - F(t) = -\int_0^t \frac{dF}{d\tau} d\tau \quad (57)$$

5. **Extract Precision:**

$$\Pi(t) = [\text{var}(\varepsilon(t))]^{-1} \quad \text{where } \varepsilon = o - g(\mu) \quad (58)$$

6. **Solve for  $\Omega$ :**

$$\Omega \approx \frac{C(t)}{\ln[\Pi(t) \cdot \Omega_{\text{ref}}]} \quad (59)$$

7. **Identify  $C_{\text{crit}}$ :**

$$C_{\text{crit}} = \text{median}\{C(t_k) : \text{rupture at } t_k\} \quad (60)$$

8. **Fit Memory Timescale:**

$$\tau_m : \langle R(t)R(t + \Delta t) \rangle \propto e^{-\Delta t/\tau_m} \quad (61)$$

## 10.2 Cross-Domain Omega Values

Table 2: Observed  $\Omega$  values across systems (from simulations)

System	$\Omega$ Range	Regime	Behavior
Ocean waves	1.3-1.7	Balanced	Predictable with variation
Hurricanes	1.2-2.0	Balanced-High	Structured chaos
Neural development	0.5-2.5	Low-Balanced	Progressive stabilization
Bee colonies	1.5-2.5	Balanced-High	Adaptive foraging
Child development (stages)	0.5 $\rightarrow$ 2.0	Low $\rightarrow$ Balanced	Increasing abstraction
Mathematical life	0.3-4.0	Full range	Regime exploration

### Observed Pattern:

- Physical systems (waves, weather):  $\Omega \approx 1.2 - 2.0$  (balanced exploration-exploitation)
- Biological systems (learning, development):  $\Omega$  increases with maturity (rigidity  $\rightarrow$  fluidity)
- Cognitive systems: Dynamic  $\Omega$  adjustment based on uncertainty

# 11 Master Equations

## 11.1 Unified Dynamics

### Complete FEP-CRR Dynamical System

$$\begin{aligned}
 \frac{dx}{dt} &= \underbrace{-\frac{\partial F}{\partial x}}_{\text{FEP Inference}} + \underbrace{\int_0^t \phi(\tau) e^{C(\tau)/\Omega} K(t-\tau) d\tau}_{\text{CRR Regeneration}} + \underbrace{\sum_i \rho_i(x) \delta(t-t_i)}_{\text{Ruptures}} \\
 C(x, t) &= \int_0^t L(x, \tau) d\tau = - \int_0^t \frac{dF}{d\tau} d\tau \\
 \Pi(t) &= \frac{e^{C(t)/\Omega}}{\Omega} \\
 \text{Rupture:} & \text{ if } F > F_{\text{thresh}} \text{ or } C > C_{\text{crit}} \\
 \text{Policy: } \pi^* &= \arg \min_{\pi} \left[ \int_t^T G(\pi, \tau) d\tau \right] \\
 \text{where } G(\pi) &= F(t|\pi) - \frac{C_{\text{expected}}(t|\pi)}{\Omega}
 \end{aligned} \tag{62}$$

## 11.2 Decomposed Equations by Operator

Table 3: Core operator equations

Operator	Equation
Coherence	$C(x, t) = \int_0^t L(x, \tau) d\tau$ where $L = -dF/dt$
Rupture	$\delta(t - t_0)$ triggered when $\ \nabla_x F\ ^2 > \text{threshold}$
Regeneration	$R[\chi] = \int_0^t \phi(\tau) \cdot \Pi(\tau) \cdot K(t - \tau) d\tau$ where $\Pi = e^{C/\Omega}/\Omega$
Precision	$\Pi(t) = [\text{var}(\varepsilon)]^{-1} = e^{C(t)/\Omega}/\Omega$
Expected Free Energy	$G(\pi) = -I[s; o \pi] + \mathbb{E}[-\ln P(o C)]$
Policy Selection	$\pi^* = \arg \max_{\pi} [\Omega \cdot \text{Epistemic} + (1/\Omega) \cdot \text{Pragmatic}]$
Markov Blanket	Maintained when $L > 0$ (coherence builds boundary)
Memory Depth	$\tau_{\text{eff}} = \int_0^t e^{C(\tau)/\Omega} d\tau$

## 12 Thermodynamic Consistency

### FEP: Non-Equilibrium Steady State

At NESS, system minimizes path-dependent free energy:

$$\mathcal{S}[q] = \int_0^T \left[ \frac{1}{2} \left\| \frac{dq}{dt} \right\|^2 + F(q, t) \right] dt \quad (63)$$

Optimal path:  $q^*(t) = \arg \min \mathcal{S}[q]$

### CRR: Action Principle

CRR dynamics derivable from action:

$$\mathcal{S}_{\text{CRR}}[x, C, R] = \int_0^T [\mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{coherence}} + \mathcal{L}_{\text{regen}}] dt \quad (64)$$

Where:

$$\mathcal{L}_{\text{coherence}} = -L(x, t) + \lambda \left( C - \int_0^t L d\tau \right) \quad (65)$$

$$\mathcal{L}_{\text{regen}} = -R[\chi] \cdot \frac{dx}{dt} \quad (66)$$

### Energy Conservation at Rupture:

$$\Delta H|_{t_0} = \underbrace{\rho(x(t_0))}_{\text{Rupture Energy}} + \underbrace{\int_0^{t_0} R[\chi] \cdot dx}_{\text{Accumulated Work}} \quad (67)$$

**Interpretation:** Discontinuous jumps conserve energy through history-weighted work terms. Ruptures are thermodynamically consistent.

## 13 Testable Predictions

### Empirical Predictions from FEP-CRR Mapping

#### Prediction 1: Universal Precision-Coherence Relation

$$\ln \Pi(t) = \frac{C(t)}{\Omega} + \text{const} \quad (68)$$

*Test:* Measure neural spike precision and entropy across learning  $\rightarrow$  should be exponentially related.

#### Prediction 2: Omega Scales with Metabolic Rate

$$\Omega_{\text{system}} \propto \frac{k_B T_{\text{eff}}}{\langle F \rangle} \quad (69)$$

*Test:* Compare  $\Omega$  across species with different metabolic rates.

#### Prediction 3: Rupture Size Power Law

$$P(\Delta C) \propto (\Delta C)^{-3/2} \exp(-\Delta C/\Omega) \quad (70)$$

*Test:* Measure discontinuous transitions (neural avalanches, earthquakes, phase changes)  $\rightarrow$  should follow Kramers statistics.

#### Prediction 4: Cross-Scale Coherence Synchronization

$$C^{(i+1)}(t) - C^{(i)}(t) = \text{const} \quad (71)$$

*Test:* Measure entropy across cortical layers  $\rightarrow$  should maintain constant offsets.

#### Prediction 5: Learning Rate Scaling

$$\eta_{\text{optimal}} = \frac{\Omega}{\langle C \rangle} \quad (72)$$

*Test:* Optimal learning rates should decrease with accumulated coherence (explains age-related learning slowdown).

## 14 Quick Reference: Symbol Glossary

### FEP Symbols

- $F$ : Variational free energy
- $\Pi$ : Precision (inverse variance)
- $\varepsilon$ : Prediction error
- $Q(\mu)$ : Recognition density
- $P(o, \mu)$ : Generative model
- $G(\pi)$ : Expected free energy
- $\mu$ : Internal states
- $o$ : Observations
- $s, a$ : Sensory, active states
- $\eta$ : External states

### CRR Symbols

- $C(x, t)$ : Coherence functional
- $L(x, t)$ : Memory density
- $\Omega$ : Temperature parameter
- $R[\chi]$ : Regeneration operator
- $\delta(t - t_i)$ : Rupture event
- $\phi(x, \tau)$ : Historical field
- $K(t - \tau)$ : Memory kernel
- $\tau_m$ : Memory timescale
- $C_{\text{crit}}$ : Rupture threshold
- $\rho_i(x)$ : Rupture amplitude

## 15 Visualization Guide

### 15.1 Reading the Diagrams

#### Figure 1: State Correspondence

- Shows parallel between FEP inference loop and CRR temporal flow
- Free energy minimization  $\leftrightarrow$  Coherence accumulation
- Prediction errors  $\leftrightarrow$  Local disorder

#### Figure 2: Omega-Precision Spectrum

- Horizontal axis:  $\Omega$  (temperature) and  $\Pi$  (precision)
- Vertical axis: Epistemic (exploration) vs Pragmatic (exploitation) value
- Shows crossover from rigid to fluid behavior

#### Figure 3: Exploration-Exploitation Landscape

- Phase diagram showing behavioral regimes
- Low  $\Omega$ : Past-dominated (crystalline structure)
- High  $\Omega$ : Future-open (gaseous exploration)

#### Figure 4: Markov Blanket as Coherence Boundary

- Coherence gradient creates statistical separation
- High  $|\nabla C|$ : Strong boundary (Markov blanket maintained)
- Low  $|\nabla C|$ : Weak boundary (system merges with environment)

#### Figure 5: Hierarchical CRR

- Multi-scale coherence fields with cross-level coupling
- Ruptures propagate up/down hierarchy
- Higher levels provide scaffolding for lower levels

## 16 Usage Notes

### How to Use This Cheat Sheet

#### For Theorists:

- Use equations to translate between FEP and CRR formalisms
- Master equations (Section 8) provide complete dynamics
- Parameter extraction (Section 7) bridges theory and data

#### For Experimentalists:

- Testable predictions (Section 9) suggest experiments
- Parameter identification (Section 7.1) guides data analysis
- Cross-domain table (Section 7.2) provides reference values

#### For Simulators:

- Implement unified dynamics (Section 8.1)
- Adjust  $\Omega$  to explore regimes (Table in Section 4)
- Use thought bubble phenomenology to interpret behavior

#### Caveats:

- These are working conjectures, not proven theorems
- Some mappings are approximate (e.g.,  $\Pi \approx e^{C/\Omega}/\Omega$ )
- Parameter values are system-dependent
- Rigorous proofs still needed for many correspondences

*This is a living document. Corrections and refinements welcome.  
Join the exploration at: <https://alexsabine.github.io/CRR/>*