Euler's Identity at the Markov Blanket Boundary

Where Imaginary Meets Real in the CRR Temporal Grammar

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Abstract

The Coherence-Rupture-Regeneration (CRR) framework describes temporal dynamics through three operators: coherence accumulation toward a threshold, rupture at that threshold, and regeneration weighted by historical coherence. For systems with Z_2 symmetry (binary phase transitions), the threshold is Omega = 1/pi, yielding precision = pi. We show that at the rupture moment, all five constants of Euler's identity (e, pi, i, 1, 0) are structurally present, that the rupture operation *is* the Z_2 group element $e^{ipi} = -1$, and that Gelfond's constant $e^{pi} = (-1)^{-i}$ emerges as the real number produced when the internal model crosses the Markov blanket boundary into external reality. These are not analogies. They are algebraic identities within the CRR formalism.

1. The CRR Operators

The CRR temporal grammar (Sabine, 2024-2025) posits three operators governing temporal processes across scales:

Coherence. C(x,t) = Integral from 0 to t of L(x,tau) d(tau), where L is the local accumulation rate. C integrates historical constraint toward a capacity threshold Omega.

Rupture. delta(t - t_0) fires when C reaches Omega. A discrete, instantaneous transition. The ontological present.

Regeneration. R(x,t) = Integral of phi(x,tau) . exp(C(x,tau)/Omega) . Theta(t - tau) d(tau), normalised by Z(t). Past states contribute to reconstruction weighted by their coherence at the time they occurred, not by recency.

The regeneration kernel exp(C/Omega) is the distinctive claim: a moment of high coherence persists in the regeneration integral regardless of how long ago it occurred. This is coherence-weighted memory rather than recency-weighted (exponential forgetting) memory, and predicts phenomena such as long-dormant skill retention, delayed trauma surfacing, and the non-Markovian character of biological memory.

2. The Euler Calibration

At rupture, C = Omega by definition. The regeneration weight at a rupture moment is therefore:

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exp(C/Omega) = exp(Omega/Omega) = exp(1) = e
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This holds for every Omega. The ratio between a rupture moment's weight and a zero-coherence moment's weight is always exactly e, regardless of the system's capacity. This is the Euler calibration: e is the fundamental constant of CRR, the fixed significance ratio at threshold.

3. Omega-Symmetry: Why Precision Is pi

The CRR framework predicts that the threshold Omega is determined by the phase-space symmetry of the system. For the two canonical symmetry classes:

Z_2 (binary flip, half-cycle): phase = pi radians, Omega = 1/pi, precision = 1/Omega = pi

SO(2) (full rotation): phase = 2pi radians, Omega = 1/2pi, precision = 1/Omega = 2pi

The pi in 'precision = pi' is not notation borrowed from the Free Energy Principle. It is the geometric constant arising from the Z_2 phase structure. A half-turn to rupture is pi radians. The precision of a Z_2 system *is* pi, by geometric necessity.

4. Euler's Five Constants at the Z_2 Rupture

At the rupture moment of a Z_2 CRR system, all five constants of Euler's identity are structurally present:

e = exp(C/Omega) = exp(1) at rupture (the regeneration weight)

pi = precision = 1/Omega (the accumulated phase at threshold)

i = the imaginary unit, representing the internal model (generative model, brain-side of the Markov blanket) meeting external reality (world-side)

1 = C/Omega, the normalised coherence ratio at threshold

0 = C resets to zero after rupture (Definition 4 of the CRR formalism)

The Z_2 group has two elements: {+1, -1}. The nontrivial element is -1. And e^{ipi} = -1. The rupture operation *is* the Z_2 group element expressed as a complex exponential. Euler's identity is not analogous to CRR rupture. It *is* the algebraic expression of the Z_2 rupture.

5. Gelfond's Constant: When Imaginary Meets Real

Gelfond's constant is e^{pi}, approximately 23.14. It equals $(-1)^{-i}$. The derivation is elementary:

$$(-1) = e^{ipi}$$
$$(-1)^{-i} = (e^{ipi})^{-i}$$
$$= e^{ipi \cdot (-i)}$$
$$= e^{-i^2 pi}$$
$$= e^{pi} \quad [\text{since } -i^2 = -(-1) = 1]$$

In CRR terms: (-1) is the Z_2 rupture element (the flip at the blanket boundary). Raising it to the power (-i) means: take the imaginary operation and invert it, crossing from internal model (imaginary) to external reality (real). The result is a real transcendental number, e^{pi}. Gelfond's constant is the blanket crossing expressed as a number.

Within the CRR formalism, e^{pi} = exp(1/Omega) for Z_2 systems. This is the *un-normalised* weight of a rupture moment, measured in raw precision units rather than the normalised C/Omega ratio. The Euler calibration gives exp(1) = e (the normalised weight). The raw precision weight gives exp(pi) = e^{pi}. Two faces of the same rupture.

6. CRR in the Complex Plane

If we represent CRR states in the complex plane with the real axis as external states (the world, Omega, measurable reality) and the imaginary axis as internal states (the generative model, accumulated C, the brain's prediction), then one Z_2 CRR cycle traces a semicircle:

Start: z = $e^{i.0}$ = +1, fully coherent with reality.

Coherence: z traces the upper semicircle (phase 0 to pi). The internal model accumulates, diverging from the real axis.

Rupture: z = e^{ipi} = -1. Maximum divergence. Inside matches outside in magnitude (|C| = |Omega|) but is opposite in sign (the Z_2 flip). This is the blanket boundary moment.

Regeneration: z traces the lower semicircle (pi to 2pi), reconstructing toward +1 using coherence-weighted memory.

Return: z = $e^{i.2pi}$ = +1. A new cycle begins. This full rotation is SO(2). The Z_2 rupture is the halfway point.

This yields a key structural insight: Z_2 is the square root of SO(2) in the group-theoretic sense. $(-1)^2$ = +1. Two Z_2 flips compose to one SO(2) rotation. Two ruptures make one full cycle. The relation $e^{ipi} . e^{ipi}$ = $e^{i.2pi}$ = +1 is this composition expressed exponentially.

7. Precision Inside the Regeneration Kernel

The regeneration weight exp(C/Omega) can be rewritten. Since Omega = 1/pi for Z_2 systems, we have C/Omega = C . pi. Therefore:

$$\exp(C/Omega) = \exp(C \cdot precision) = \exp(C \cdot pi)$$

Precision is not multiplied in from outside. It is already the exponent inside every evaluation of the regeneration kernel. Every time a Z_2 CRR system computes exp(C/Omega) at any moment in its history, it is computing exp(C . pi). The constant pi is doing the work inside every regeneration integral, at every scale.

The dynamic range of the memory kernel for any CRR system is [1, e], from C = 0 (weight = 1) to C = Omega (weight = e). But the raw, un-normalised weight at rupture is exp(1/Omega), which for Z_2 is exp(pi) = e^{pi}. For SO(2) it is exp(2pi) = e^{2pi}. The ratio between them is e^{2pi}/e^{pi} = e^{pi}. The constant recurs at every level.

8. Connection to the Free Energy Principle

In the Free Energy Principle (Friston, 2010), precision (conventionally denoted Pi) is the inverse variance of a probabilistic belief: Pi = 1/$sigma^2$. CRR proposes that Omega = $sigma^2$ = 1/precision, making the identification Omega = 1/Pi direct.

For Z_2 biological systems, this gives Pi = pi: the precision of the system's beliefs at the Markov blanket boundary is the geometric constant pi. This is not a coincidence of notation. If the CRR Omega-symmetry prediction is correct, then the pi in Friston's precision is the same pi as in Euler's identity, arising from the Z_2 phase structure of biological state transitions. CRR provides the temporal dynamics that the FEP presupposes: *when* beliefs update (C reaches Omega), and *how* history shapes reconstitution (the exp(C/Omega) kernel). Rupture is Bayesian model reduction. Regeneration is posterior reconstruction.

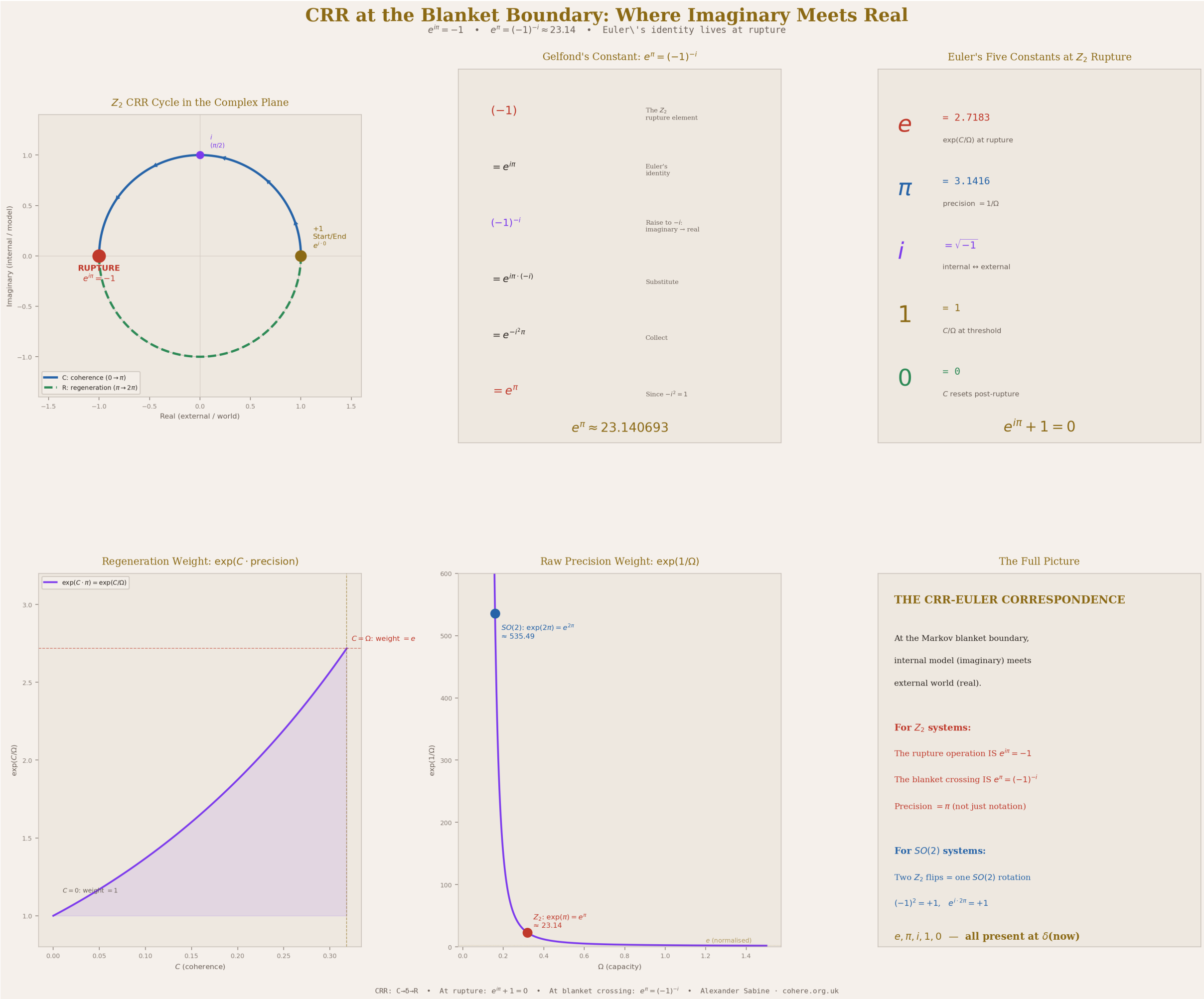

Figure 1. *The CRR-Euler correspondence. Top left: a Z_2 CRR cycle in the complex plane, with coherence tracing the upper semicircle (0 to pi) and regeneration the lower (pi to 2pi). Top centre: derivation of Gelfond's constant $e^{pi} = (-1)^{-i}$. Top right: the five constants of Euler's identity at the Z_2 rupture moment. Bottom left: the regeneration weight exp(C . pi) as a function of coherence. Bottom centre: the raw precision weight exp(1/Omega), showing e^{pi} at the Z_2 threshold and e^{2pi} at SO(2). Bottom right: summary of the correspondence.*

9. Summary of Claims

The following are algebraic identities within the CRR formalism, not analogies:

(i) At Z_2 rupture, all five constants of Euler's identity are structurally present: e (regeneration weight), pi (precision), i (internal/external distinction), 1 (normalised threshold), 0 (post-rupture reset).

(ii) The Z_2 rupture operation is $e^{ipi} = -1$. The nontrivial group element, expressed as a complex exponential.

(iii) Gelfond's constant $e^{pi} = (-1)^{-i}$ is the real number that results when the internal model's rupture crosses the Markov blanket into external reality.

(iv) Precision = pi for Z_2 systems, by geometric necessity (the half-turn to rupture is pi radians), not by notational convention.

(v) Z_2 is the square root of SO(2): two flips compose to one full rotation. $e^{ipi} . e^{ipi} = e^{i.2pi} = +1$.

(vi) The regeneration kernel exp(C/Omega) = exp(C . pi) for Z_2 systems. Precision is already inside every evaluation of the regeneration integral.
