The Boundary is a Rate: Coherence–Rupture–Regeneration as a Temporal Physics of Emptiness Realisation

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Abstract. Sandved-Smith, Fields, Doctor, Laukkonen and Hohwy argue that any system which persists must self-evidence, gathering evidence for its own generative model, but that one belief is provably beyond reach: evidence that the system is *separate* from its environment. Because no finite system can measure the entanglement entropy across its own boundary from inside, separability is permanently unevidenceable, and they read the realisation of emptiness as the embodied recognition of this impossibility. We take their result to be correct and do not contest it. We ask what survives it when the boundary is read not as a static holographic screen but as a *rate*: the moving locus, in the Coherence–Rupture–Regeneration (CRR) framework, at which accumulated Fisher information saturates the Cramér–Rao bound, $C \cdot \Omega = 1$. Embedding CRR's three equations as finite operational surrogates in the quantum-reference-frame (QRF) formalism the original argument employs, we find that the rupture instant is itself screen-inducing, so CRR inherits the impossibility rather than escaping it; that the point at which CRR fails the formalism's unitarity requirement is, on a measurement-theoretic reading, a signature rather than a defect; that the forbidden spatial correlation is robustly *bounded* by, though not identical to, an accumulated temporal quantity; and that the convergence of many finite systems produces a macroscopic collective now through a synchronisation transition we establish by finite-size scaling. The contribution is a relocation, not a refutation: emptiness moves from an epistemic limit on measurement to an ontological claim about a present with no content of its own, met by the selectively weighted past of every other finite system. All results are consistency demonstrations under a chosen formalism, not empirical findings.

Keywords: self-evidencing; emptiness; quantum reference frames; holographic screens; free energy principle; Fisher information; Cramér–Rao saturation; synchronisation.

1 Introduction

To persist is to keep confirming a model of what one is and what surrounds one. In the free-energy principle (FEP), a system that endures self-evidences: it minimises the surprisal of its sensory states and thereby accumulates evidence for its own generative model. Sandved-Smith and colleagues [1] identify a single belief this process can never reach. To confirm that one is *separate*—bounded off as an entity distinct from one's environment—would require quantifying one's entanglement with everything one is not, and a finite system provably cannot measure the entanglement entropy across its own boundary from within. Separability is therefore not merely unconfirmed but structurally unevidenceable; and they propose that awakening, as the stable realisation of emptiness, is the embodied recognition of exactly this impossibility.

We find the argument compelling and, on our reading, its central result correct. What we examine is a picture it leaves intact: that behind the unmeasurable boundary there remains a determinate environment, a far side with which one is entangled but whose correlation one can never quantify. Coherence–Rupture–Regeneration (CRR) is a temporal framework in which bounded systems accumulate evidence, reach the limit of a regime, and reconstruct from their own history. Here we turn it on the boundary the impossibility theorem concerns, and ask what emptiness leaves standing once that boundary is read as a rate rather than a surface.

The claim in one sentence. We agree the impossibility theorem holds, inherit it rather than escape it, and then make a single ontological move: behind the unmeasurable boundary there is no determinate far side—only the kernel-weighted past of every other finite system, met at a present that has no content of its own.

This paper is self-contained. Section 2 sets out CRR; Section 2.1 shows that its rupture is $\mathbb{Z}_2$ by construction, and Section 2.2 gathers, under a strict discipline, the contemplative and process traditions whose language matches specific features of the geometry. Section 3 reconstructs the impossibility argument. Section 4—the technical core—embeds CRR in the QRF formalism and states what that embedding forces, forbids, and over-determines. Sections 5–8 draw out the ontological consequences; Section 9 states pre-registered predictions; Sections 10–11 conclude. Appendix A fixes notation and documents the operational surrogate suite; Appendix B lists the deterministic script `boundary.py` in full.

2 Coherence–Rupture–Regeneration

CRR describes any persisting process—a wound healing, a belief stabilising, an attractor forming—through the accumulation, saturation, and rebuilding of coherence. We use three notions from information geometry, defined here. A system's momentary state is a point on a statistical manifold; the Fisher–Rao speed $L \geq 0$ is the local rate at which it accrues distinguishable structure. *Coherence* accumulates as the integral of that speed along the trajectory,

$$C(x,t) = \int_0^t L(x,\tau)\, d\tau, \tag{1}$$

which, under the path-integral formulation of the FEP, is accumulated arc length along the information-geometric trajectory. The content of the present is, by this definition, exactly what has been integrated over the past; the upper limit t contributes nothing of its own—an observation, trivial as a statement about an integral, that becomes load-bearing in Section 5.

Coherence cannot accumulate without bound. The Cramér–Rao bound sets the smallest variance any unbiased estimator can attain from given information; when accumulated coherence saturates it, the process *ruptures*—a discrete event of zero temporal extent, carrying one unit of mass on the boundary between the settled past and the regenerated future,

$$\delta(\text{now}) \quad \text{when} \quad C\cdot\Omega = 1. \tag{2}$$

The dimensionless product $C\cdot\Omega$ gives (2) the same algebraic form as the Heisenberg relation and the thermodynamic uncertainty relation; the kinship is qualified in Section 4. After rupture the process *regenerates* from its own history through a coherence-weighted memory kernel,

$$R(x,t) = \int \varphi(x,\tau)\, e^{C(x,\tau)/\Omega}\, \Theta(t-\tau)\, d\tau, \tag{3}$$

in which the Heaviside step Θ enforces that only the settled past contributes, and $e^{C/\Omega}$ is the maximum-entropy weighting consistent with a constraint on $\langle C \rangle$. Regeneration *prehends* the past; it does not replay it.

The single parameter is fixed by geometry, not fitted. By Čencov's uniqueness theorem the Fisher–Rao metric is, up to scaling, the only metric on a statistical manifold invariant under sufficient statistics; naming the manifold fixes its geodesic extent ℓ, and we take $\Omega = 1/\ell$. A bistable process inhabits the Bernoulli interval of geodesic diameter π, giving $\Omega_{\mathbb{Z}_2} = 1/\pi$ and rupture coherence $C^* = \pi$; a rotational process inhabits the circle of circumference 2π, giving $\Omega_{SO(2)} = 1/2\pi$ and $C^* = 2\pi$. The ratio $C^*_{SO(2)}/C^*_{\mathbb{Z}_2} = 2$ exactly. The same geometry fixes the variability of rupture timing: the coefficient of variation of the inter-rupture interval is $\mathrm{CV} = 1/2\pi \approx 0.159$ for a bistable system and $1/4\pi \approx 0.080$ for a rotational one—a parameter-free signature returned to in Section 9. Finally, Ω acts as a temporal light cone: small Ω peaks the memory kernel (rigid, repetitive reconstruction), large Ω flattens it (flexible, exploratory).

This last property is where CRR first meets a second vocabulary exactly. Doctor and colleagues [4] place *care*—in their phrasing, the capacity to exert effort toward preferred states—at the root of intelligence, scaled by an agent's *cognitive light cone*, the reach of what it can hold as a goal, and define an agent's *stress* as the gap between its current and optimal state. In CRR that gap is a definite quantity: it is $1 - C\cdot\Omega$, the distance the inferential map still has to travel before it saturates the territory (Section 6). Doctor's stress, care, and light cone are not analogies for Ω and its dynamics; they are three faces of the one geometric object, and we shall use them as such.

In a nutshell

The mathematics. Coherence C accumulates as (1); at $C\cdot\Omega = 1$ a discrete cut falls between past and future, (2); regeneration (3) rebuilds from history, weighting the most coherent moments most. $\Omega = 1/\ell$ is fixed by geometry, with $\Omega_{\mathbb{Z}_2} = 1/\pi$, $\Omega_{SO(2)} = 1/2\pi$, ratio 2. The map–territory gap $1 - C\cdot\Omega$ is Doctor's *stress*. *The reading.* A system builds structure until it can build no more, breaks, and remakes itself from what it has been, leaning hardest on the moments that mattered most. Care sets what it reaches for; stress is how far it still is; the light cone is how much past and future it can hold at once—one invariant, Ω, in three roles.

2.1 The rupture is $\mathbb{Z}_2$ by construction

The two symmetry classes read easily as alternatives—a system is bistable *or* rotational. For the boundary the impossibility theorem concerns, however, their relation is not parallel but nested, and the geometry makes the nesting exact. Under the Fisher–Rao amplitude embedding $p \mapsto (\sqrt{1-p}, \sqrt{p})$, the Bernoulli manifold traces a geodesic arc of length exactly π: the geodesic coordinate $\phi(p) = 2\arcsin\sqrt{p}$ sweeps $0 \to \pi$ as p runs $0 \to 1$. The bistable manifold is therefore not merely as long as half the $SO(2)$ circle of circumference 2π; it *is* a semicircle of that circle (Figure 1).

Two uses of one geometry must accordingly be kept apart. The *shape of the question*—corridor or loop—fixes ℓ, hence Ω, and varies by system. The *two aspects of every rupture*—the continuum that holds, the cut that falls—never vary. We reserve $\mathbb{Z}_2$ for the structure of the cut and $SO(2)$ for the continuum. The circle supplies duration; the crossing supplies the cut; they compose rather than compete. This also settles why the factor of two is forced rather than fitted: the circle is the Bernoulli arc completed into a full turn, so $C^*_{SO(2)} = 2\,C^*_{\mathbb{Z}_2}$ follows from the doubling alone.

The nested configuration has a long pedigree, which we note because the geometry underwrites it without depending on any part of it. In the 莊子, Zhuangzi names the still centre of a turning ring the *pivot of the Way*, 道樞: the pivot grasps the centre of the ring and thereby responds without end [7]. An unmarked centre on a ring, belonging to no po-

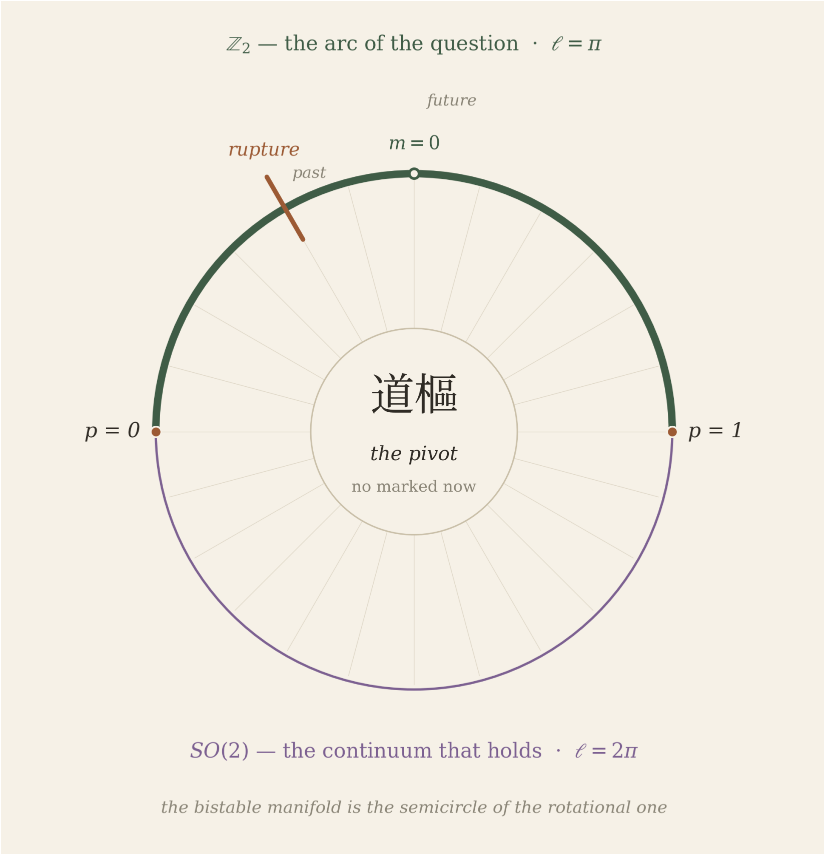

Figure 1. Two uses of one geometry. The circle is the $SO(2)$ continuum—no preferred now, $\ell = 2\pi$. Its upper arc is the $\mathbb{Z}_2$ semicircle, the shape of the bistable question, endpoints $p = 0$, $p = 1$, midpoint $m = 0$; $\ell = \pi$. The contentless centre is the present with no content of its own; the mark is the single rupture, the $\mathbb{Z}_2$ cut between past and future. In the 莊子, the still centre of a turning ring is the *pivot of the Way* (道樞): it grasps the centre of the ring and thereby responds without end.

sition yet meeting every one, is the $SO(2)$ continuum with a contentless now at its hub; the Daoist adds only what the geometry implies, that responsiveness is a property of *not* being fixed at any point. In the Madhyamaka the two poles are *prajñā* and *upāya*: wisdom, which sees the empty, groundless nature of things, and skilful means, the act that meets a particular situation. The continuous circle is the figure of *prajñā*, the open ground; the discrete $\mathbb{Z}_2$ crossing, a directed cut from a settled past to an opening future, is the figure of *upāya*; their inseparability is the classical claim that wisdom and method are never complete alone. In the Whiteheadian register the circle is the concrescing process—the becoming with no determinate datum while underway—and the rupture is *satisfaction*, the moment the private process perishes into a public fact the next occasion inherits.

2.2 A convergence across contemplative and process traditions

A commentary that let this become a catalogue of traditions which "also valued the present" would be worth little. We hold a strict test: a tradition earns a place here only where its language matches a *specific* structural feature of the framework—an equation, a geodesic extent, a rate—and not merely the mood of presence. By that test four namings are exact (Figure 2), and two traditions further supply the ethics the rate implies.

Emptiness as enabling, not privative. The present's want of content is not a poverty but a capacity: the one locus at which every settled past can arrive, none crowding out another. This is Nāgārjuna's move in the *Mūlamadhyamakakārikā*: for the one for whom emptiness is possible, everything is possible (24.14), so that emptiness is the condition of function, not its negation; and to call a thing empty is to say it arises dependently, which is itself the middle way (24.18) [6]. The world, on this reading, is not a domain of being but of ceaseless becoming—things do not statically exist, they occur, as a melody exists only in the passing of its tones.

The vacated centre. Lurianic Kabbalah names the same structure cosmologically. In *tzimtzum*, the infinite contracts to open a *ḥalal ha-panuy*, a vacant space, into which creation can beam—a single locus of simultaneous presence and absence [9]. That is the contentless now to the letter: the same fact that empties the present is the fact that lets every past into it.

The pulse. From Ezekiel's living creatures the tradition draws *ratzo va-shov*, running and returning: a surge to a limit and a withdrawal, the fundamental beat of vitality. It is Equations (2)–(3) read as a pulse—the rupture cycle as a relaxation oscillator—and, in the register of the body, the breath: inhalation as C, the turn as δ, exhalation as R.

Owner of the moment. Sufism supplies the distinction the framework needs for agency. The wayfarer is *ibn al-waqt*, the son of the moment—subject to the moment's rate, as no system may set its own Ω. But the realised one, Ibn 'Arabī argues, is rather *ṣāḥib al-waqt*, the owner of the moment: not master of the rate, but of the kernel—of φ, what is drawn upon when the past is taken up again [8]. One cannot choose Ω; one can choose what to prehend.[1]

Two traditions do more than name; they supply the *ethics* the rate implies, and both are Hershock's [5]. Saturation, $C\cdot\Omega\to 1$, is *phala* in his gloss—fruit ripe to the point of bursting, the concentrated expression of accumulated conditions. Its far face is opportunity, the opening through which redirection enters: karma is not a ledger etched in stone but an ongoing improvisation, continually reopening for reshaping—which is exactly a past whose content is closed and whose causal contribution is live. And when many finite systems couple, the shared present they compose is Hershock's *karmic cohort* given a dynamical criterion (Section 8): below a critical coupling only private nows survive; above it, a present held in common, whose responsibility is neither one's own nor another's.

We rest no formal weight on any of these correspondences. The single fact of information geometry—that the bistable manifold is the semicircle of the rotational one—is what each of them can be seen as pointing at. The traditions did not derive the rate; they met it, in their own idioms, and named what they met.

In a nutshell

The mathematics. Four exact matches: enabling emptiness (Nāgārjuna 24.14) for the non-privative present; the vacated centre (*tzimtzum*) for the contentless now; *ratzo va-shov* for the rupture–regeneration pulse; *ṣāḥib al-waqt* for agency over φ but not Ω. Hershock's *phala* and karmic cohort name saturation and the collective now. *The reading.* The now holds nothing so that it can hold everything—one claim, stated in information geometry and, long before, in traditions that could not have shared

[1] The pairing is not idle: scholars have noted a kinship between the treatment of metaphysical time in Rūmī and in Blake—two poets for whom the fully met moment opens onto eternity rather than merely passing through it.

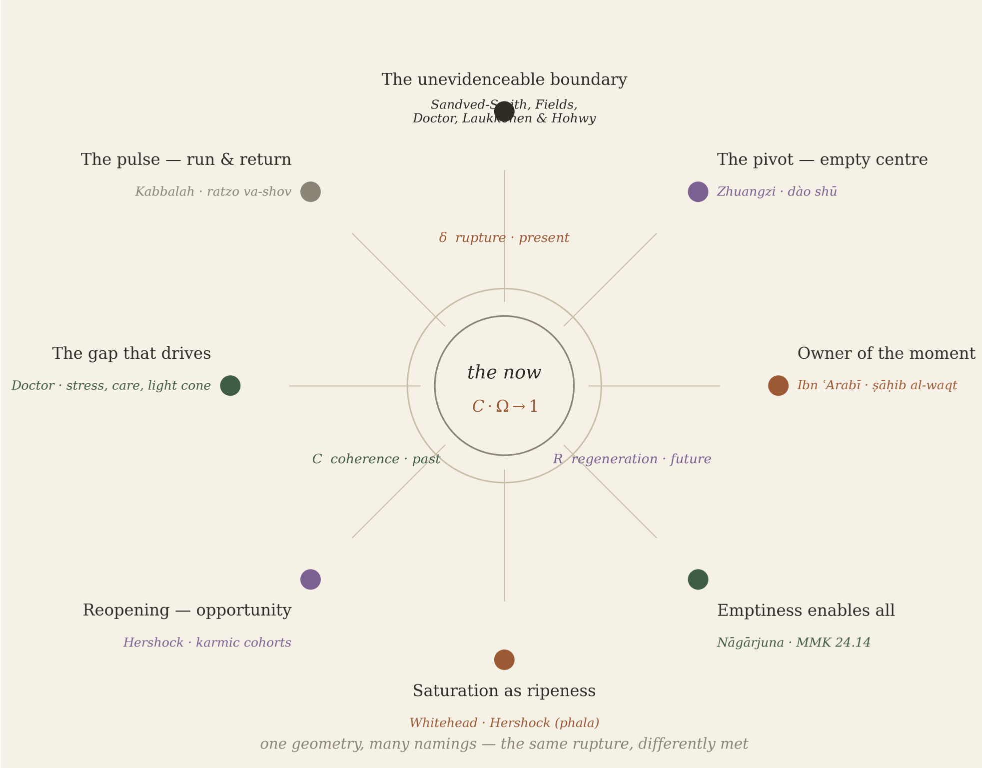

Figure 2. One geometry, many namings: the same rupture, differently met. The theorem CRR inherits is Sandved-Smith, Fields, Doctor, Laukkonen and Hohwy's unevidenceable boundary (top). Around it: Doctor's gap that drives; the ripeness and reopening Hershock reads in karma; the Kabbalistic pulse; Nāgārjuna's enabling emptiness; Zhuangzi's empty pivot; Ibn ʿArabī's owner of the moment. Each names a feature of one geometry; none is imported to prove it.

a proof. That is why the convergence is evidence of a shape in the world, not of influence between texts.

3 The unevidenceable boundary

We reconstruct the impossibility argument at full strength before reading it, because a commentary that weakened it first would be worth nothing. It proceeds in four moves. *First, the premise:* any system that persists must minimise the surprisal of its sensory states, equivalently maximise the evidence for its generative model; to exist is to self-evidence. *Second, the one unreachable belief:* among all the beliefs a self-evidencing system maintains, exactly one can never receive evidence—that there is a boundary constituting it as separate from its environment. *Third, the quantum-information ground:* confirming separateness would require measuring the entanglement entropy across one's own Markov blanket, quantifying how correlated one is with everything one is not, and recent results in the quantum-FEP programme [3] establish that no finite system can perform this measurement across its own boundary from inside. The authors formalise the belief in separation as a *structural prior* over the agent's quantum-reference-frame (QRF) deployments—the choices of measurement basis available to it—constraining every frame to respect a self/environment partition: the partition is built into the apparatus of measurement and is never among the things measured. *Fourth, the interpretive move:* if separation is a prior that can never become a posterior, then awakening, as the stable realisation of emptiness, is the embodied recognition of this impossibility. The authors mark this fourth move as interpretive, and we respect that marking.

It is worth being precise about the kind of result this is. It is an impossibility theorem embedded in a chosen formalism: its truth is the truth of what the QRF formalism forbids—a strong and honest kind of truth, but not the same kind as a measurement of the world, and the authors do not pretend otherwise. Keeping that distinction in view is what allows the CRR reading below to be a genuine response rather than a rival assertion.

4 CRR in the quantum-reference-frame formalism

This is the technical core of the paper: how the three CRR equations are made to interpret, rather than merely gesture at, the mathematics of the impossibility theorem. We follow the method the original programme itself uses, and its operational companion [2], in which the same qFEP and QRF equations are rendered as finite operational surrogates checked by positive controls and by discriminating tests that reject stronger readings. We do not test CRR against data. We embed its three equations in the QRF and holographic-screen formalism—the description of a boundary as a finite channel carrying a bounded number of distinguishable sectors—and ask what that formalism forces, forbids, or over-determines. Four findings bear on the impossibility theorem; for each we separate what the computation settles from what it can only illustrate. Every quantity named below is reproduced by the deterministic suite of Appendix B, whose claim ledger binds each figure to the equation it interprets (Appendix A).

4.1 The embedding

In the QRF picture a system A and its environment E share a boundary that acts as a holographic screen: a finite channel whose bounded sector-count carries the entanglement entropy $S(\rho_A)$, with mutual information $I(A{:}E) = 2S(\rho_A)$. The impossibility theorem is the statement that a finite A cannot read $S(\rho_A)$ across its own screen from inside. CRR does not add a term to this formalism; it re-reads the boundary. Where the QRF screen is a *surface* at which a definite partition stands, CRR's boundary is the *rate* at which coherence approaches saturation, $C \cdot \Omega \to 1$ (Eq. 2). The embedding then proceeds by giving each CRR object a finite operational surrogate inside the QRF machinery and asking what the machinery makes of it. The coherence integral (1) supplies the accumulated Fisher information that the Cramér–Rao bound of (2) constrains; the rupture (2) is tested for whether it induces a screen at all; and the regeneration kernel (3) is tested against the formalism's unitarity requirement.

4.2 The rupture is screen-inducing

The prior question is whether CRR's rate-boundary possesses a screen in the QRF sense. Constructing an explicit bipartite pure state on $A \otimes E$ and driving an internal coherence to saturation, we find that the reduced state ρ_A remains a valid density matrix—positive semi-definite and unit-trace to machine precision (4×10^{-16})—with finite von Neumann entropy at every saturation level $s \approx C \cdot \Omega$ up to and including the rupture instant, where $S(\rho_A)$ reaches its maximum of 2 bits for the four-dimensional subsystem (deterministic surrogate `fig2_screen_induction`; Appendix B). A definite self/environment partition exists *exactly* when $C \cdot \Omega \to 1$. The consequence is the opposite of an escape: CRR inherits the QRF machinery, and with it the impossibility theorem. Whatever account of emptiness

CRR offers cannot be a way around the unmeasurability the original paper proves; it must be something else.

4.3 Saturation is a genuine Cramér–Rao equality, and the prediction it forces

The condition $C\Omega = 1$ is verified as a saturated Cramér–Rao bound—the equality a maximum-likelihood estimator attains for a Gaussian family—and the bistable-to-rotational ratio of inter-rupture arc comes out as 2 to machine precision. Embedded in the holographic-screen picture, this over-determines a consequence neither framework contained alone. If $\Omega = 1/\ell$ is a geodesic extent and a screen carries a finite number of distinguishable sectors, then a rotational system must expose exactly *twice* the boundary sector-count of a bistable one: its inter-rupture arc is twice as long at fixed sector density. This prediction is a product of the encounter, and it is falsifiable. One correspondence CRR has at times stated too strongly should be qualified in the same spirit: the kinship between $C{\cdot}\Omega = 1$ and the Heisenberg relation holds as an algebraic form, not a literal identification of C, Ω with $\Delta E, \Delta t$. The dimensionless product carries the form of a saturated uncertainty relation, and that is all it carries.

4.4 The regeneration kernel fails the unitarity requirement—and why that is a signature

The quantum-FEP requires the joint system to be unitary—information conserved, the screen merely relocating it—and establishes an asymptotic equivalence between the FEP and unitarity [3]. CRR's regeneration kernel $e^{C/\Omega}$ does not pass this requirement, and the failure is informative rather than fatal. Read literally as amplification, the kernel injects of order $C^*/\Omega = \pi^2 \approx 9.9$ nats per regeneration and inflates the state norm by $e^{\pi^2} \approx 1.9 \times 10^4$, four orders of magnitude. Read as renormalised selection weights, it conserves total measure but *reduces* entropy below the input, $\Delta H = -1.34$ nats (from $\ln 12 = 2.485$ nats to 1.141 nats)—a basis change toward the highest-coherence moments rather than an information pump (surrogate `fig3_kernel_fork`; the literal "information-pump" reading is rejected). As written, CRR cannot be a closed unitary system. This is the same species of finding the original paper produces—the formalism forbidding something the equations took for granted—and it points somewhere specific. If the amplification must be paid for, writing $e^{C/\Omega}$ as drawing on an explicit free-energy budget makes CRR an open system and couples Ω, until now purely geometric, to a thermodynamic quantity. Section 6 argues that the non-unitarity is more than a bridge: read measurement-theoretically, it is the mark of a collapse, and the budget it owes is a Landauer cost.

4.5 The self-measurement obstruction, illustrated

On the same system, the obstruction the impossibility theorem proves in general can be exhibited on examples. The agent's read-out of its own boundary entropy in any measurement frame it can deploy systematically overshoots the true value $S(\rho_A) = 1.626$ bits—mean overshoot $+0.31$ bits across eight arbitrary (Haar-random) frames—coinciding with it only in the eigenbasis of ρ_A, which the agent cannot identify without already diagonalising its own state (surrogate `fig4_self_measurement`). Recovering the true entropy would demand full tomography and a stable external reference frame, precisely what a finite system lacks for its own boundary. This illustrates, and does not prove, the universal result.

In a nutshell

The mathematics. A definite screen exists at the rupture instant ($S(\rho_A) \to 2$ bits), so CRR inherits the impossibility rather than escaping it; $C\cdot\Omega = 1$ is a genuine Cramér–Rao equality; the encounter forces a falsifiable prediction (rotational screens expose twice the boundary sectors of bistable ones); and the kernel $e^{C/\Omega}$ fails unitarity—literally $+9.9$ nats and $\times 1.9\times 10^4$; renormalised, $\Delta H = -1.34$ nats. *The interpretation.* CRR does not evade the theorem; it embeds inside it and re-reads its terms. The boundary is not a surface the agent fails to measure but a rate it constitutively runs; the kernel's non-unitarity is not a bug but the signature of a measurement-like event, whose thermodynamic cost the framework independently owes.

5 The boundary as a rate

The boundary is not a thing one has but a rate one runs. The holographic screen of the original programme is static: at any instant a definite self/environment partition exists, and the theorem forbids measuring across it. CRR does not begin with a partition. What plays the role of the boundary is the locus at which coherence is about to saturate and break, and because every system carries its own Ω—its own symmetry class, its own saturation rate—there is no single screen but an interference of many rates meeting. That the boundary is co-constructed at all scales is, in CRR, not a soft phrase but a consequence of there being no single Ω in the universe.

The original claim that one cannot measure across one's boundary becomes more specific and more mechanical: the edge one cannot get behind is the front of an accumulation, and one cannot stand behind it because the standing-behind would be further accumulation. Here CRR has a claim of its own, beginning from a fact already present in (1). The coherence $C(x,t)$ is an integral over τ up to the present; content—distinguishability, accumulated Fisher information—is exactly what has been integrated over the past, and the present instant, as the upper limit of integration, carries none of its own. So the question the screen account invites—what, then, is on the far side?—receives a different answer. What is out there is not a hidden determinate environment behind a screen; it is the settled past of every other finite system, met at the contentless instant of now. A seam must not be smoothed over. A naïve reading would make the world a flat archive, all completed pasts equally present; the regeneration term (3) forbids this, for the kernel $e^{C/\Omega}$ is selective. Two systems meeting in the same now do not encounter a shared archive but each enact the other according to its own kernel and its own Ω. The past has content, but the content is prehended, never simply given—closer to the enactive tradition than to the screen formalism. There is no far side; there is only earlier, taken up selectively.

6 The coincidence of map and territory

We now reach what occurs at the only instant the boundary exists—the rupture, at which $C\Omega = 1$—and argue that the phrase *the map becomes the territory* is there a close description of Cramér–Rao saturation. A saturated estimator is a *sufficient statistic*: a function of the data carrying everything the data contain about the parameter, with no slack remaining. Below $C\cdot\Omega = 1$ there is a gap, the map still chasing the bound set by the territory's informational footprint; at saturation the gap closes to a point, the map holding exactly the territory's footprint—no more (the bound forbids it) and no less (saturation means it is reached). Sufficiency is the coincidence, and it occurs at every now, because each rupture is a fresh

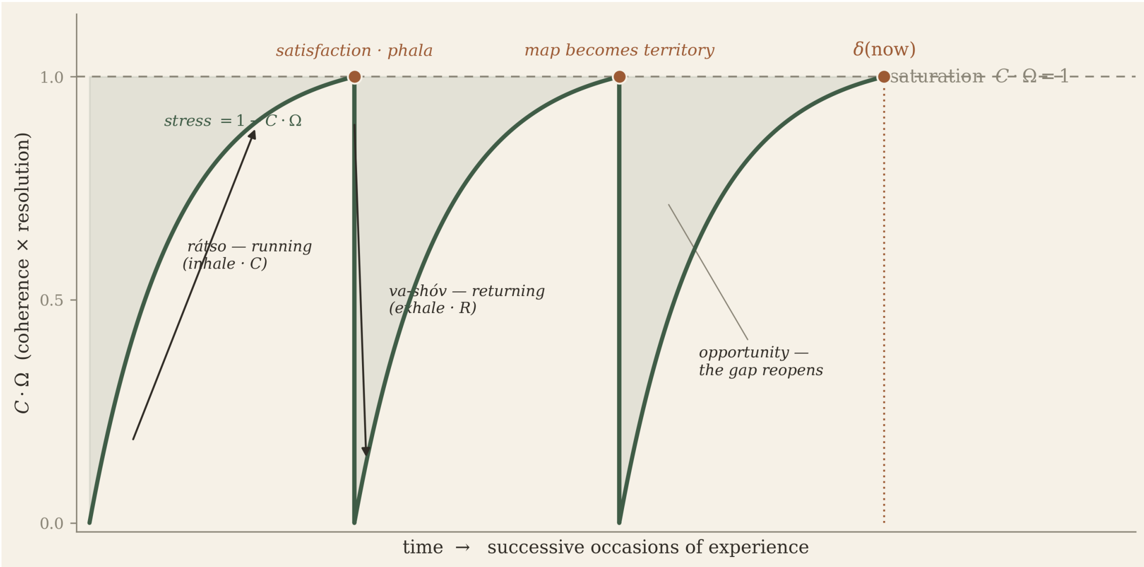

Figure 3. The boundary is re-enacted at every now. Coherence accumulates toward saturation $C \cdot \Omega = 1$ within each occasion; the shaded gap $1 - C \cdot \Omega$ is Doctor's *stress*, closing to zero at each rupture—*satisfaction* (Whitehead), the fruit ripe to bursting (*phala*, Hershock)—then reopening as *opportunity*. The rise-and-return is the Kabbalistic pulse *ratzo va-shov*; read as breath, it is inhalation, the turn, and exhalation. Deterministic surrogate `fig6_every_now`.

saturation; the boundary does not persist but is re-enacted at each δ(now) (Figure 3). In Doctor's terms the closing gap is stress falling to zero; the reopening is the next occasion's stress. That the boundary exists only momentarily is Whitehead's transition from subject to superject read as a saturation dynamics: an occasion is map-side while becoming, territory-side once become; *satisfaction* is $C \cdot \Omega = 1$, and the map does not represent the territory at saturation—it perishes into it.

This reading changes what the non-unitarity of Section 4 is. In quantum mechanics unitary evolution is the closed-system, map-side dynamics; the non-unitary part is measurement, the collapse in which a quantum map becomes classical territory. If the rupture is the instant at which map becomes territory, then the regeneration kernel *should* not be unitary—the rupture is a measurement-like event, and CRR's failure of the unitarity requirement is a signature, not a defect. The two readings of the kernel converge thermodynamically: a measurement carries a Landauer cost, and that cost is exactly the free-energy budget for Ω that Section 4 found CRR to owe. Map-becomes-territory, measurement, non-unitarity, and Landauer cost are one event described four ways. A new problem then arises, which honesty requires naming: if the rupture is a measurement it requires a reference frame, and a system measuring itself into existence at each now would be performing the very self-measurement the theorem forbids. The escape follows from Section 5: what collapses at each rupture is not the boundary but the past the system measures—the settled occasions it prehends, not its own separability. The agent never measures the entanglement entropy across its own screen (forbidden, and CRR agrees); what it measures is its prehended past (permitted, and contentful).

7 Retensing the forbidden correlation

The impossibility theorem forbids measuring $I(A{:}E) = 2S(\rho_A)$: one's correlation with the co-present complement, quantified across the boundary from inside. But the forbidden quantity is stated over a co-present *spatial* complement, and in CRR there is no co-present totality of not-self—the far side is the settled past (Section 5). This suggests a question: might the correlation that cannot be measured across the boundary be recoverable as a quantity accumulated *along the past*, retensed from a spatial into a temporal object? The accumulated coherence C is exactly such an object. We construct explicit bipartite systems—a system qubit coupled to a chaotic (GUE) bath—and at each time compute both the forbidden $S_A(t) = \frac{1}{2}I(A{:}E)$ and an accumulated $C(t) = \int_0^t L\,d\tau$ from the system's own trajectory (surrogate `fig7_retensing`). The findings are more qualified than the strongest conjecture and more interesting than its failure: the strong identity $C = I(A{:}E)$ is false—C grows monotonically while $I(A{:}E)$ oscillates back to zero—but a bound holds robustly, $C(t) \geq S_A(t)$ pointwise across every bath size $(4, 6, 8$ qubits) and all 1920 sampled points, touching only at $t = 0$, with C tracking S_A at correlation 0.997 in the fast-mixing regime where rupture lives. The forbidden spatial quantity is therefore *bounded* above by an accumulated temporal one, not identical to it. CRR does not dissolve the impossibility by computing the forbidden quantity through time; the resolution stays ontological—CRR denies that the co-present complement exists—but it is now supported by a robust bound rather than a numerical identity, and the more so because the test was built to give the identity its best chance and the identity declined.

8 Co-construction of the collective now

The boundary-as-rate reading entails that the now is not any one system's possession but is co-constructed where many systems' saturations meet. Because the rupture–regeneration cycle of (2)–(3) is a relaxation oscillator, this becomes a question in nonlinear dynamics. *Two systems.* In an explicit quantum model of two coupled subsystems, coupling makes each system's boundary formation depend on the other's state, mutually and growing with coupling strength. With $\Omega_P = 1/\pi$ (bistable) and $\Omega_Q = 1/2\pi$ (rotational), P ruptures twice as often as Q: the natural ratio is the topological 2:1. Sweeping the natural ratio and measuring the rotation number ρ, we find that at zero coupling ρ follows the natural ratio (merely co-present), while at non-zero coupling a plateau locked at $\rho = 2$ opens—an Arnold tongue, inside which neither boundary is placeable independently (surrogate `fig8_two_body`). At exactly the topological ratio the systems lock for *any* non-zero coupling ($\rho = 2.0005$ throughout), the tongue widening from a point at $\kappa = 0$ to the full swept range by $\kappa = 0.25$: CRR sits at the centre of the tongue by topology, not by tuning.

Many systems. Whether the pairwise result extends to the convergence of all finite systems is the Kuramoto problem: N rupture oscillators with a spread of natural frequencies, coupled through their contact at the shared now (surrogate `fig9_kuramoto`). We hold the result to the standard of the literature: for a Lorentzian frequency distribution of half-width γ the transition has exact critical coupling $K_c = 2\gamma$ and $r(K) = \sqrt{1 - K_c/K}$, which the simulator reproduces to within 0.015. Finite-size scaling then establishes a genuine transition: below K_c the order parameter falls as $N^{-0.47}$, consistent with the incoherent $N^{-1/2}$ (no shared now in the limit); above K_c it converges to a finite, N-independent value (a shared now), and the locked fraction rises from a few per cent to a macroscopic value, stable as N grows from hundreds to thousands. We add one correction against over-extrapolation: the pairwise 2:1 resonance governs weak-coupling cross-cluster locking, but as coupling strengthens a common effective frequency and global 1:1 entrainment override it. The two-body and many-body results are related but distinct regimes; we do not claim the 2:1 propagates to the collective. This pairwise structure echoes a finding of the companion study [10], where coordination between two channels was likewise a two-body phenomenon that scales by pairwise composition rather than by adding channels within a single agent.

9 A pre-registered prediction: the breath

The breath, used in Section 2.1 to fix the two symmetry classes, carries the framework's cleanest contact with measurable data. We state the predictions and do not test them here, so that the formal status of the consistency demonstrations above is preserved; the discipline is that of pre-registration. *The coefficient-of-variation prediction.* The respiratory cycle is an autonomous limit-cycle oscillator whose phase runs a closed loop; as a system its question is rotational, so it takes $\Omega = 1/2\pi$ and ruptures once per cycle, and the parameter-free prediction of Section 2 is that the coefficient of variation of the inter-breath interval is $\mathrm{CV} = 1/4\pi \approx 0.080$. The clean test is the most autonomous condition available—quiet NREM sleep or a CO_2-clamped protocol—where cortical override is minimal; that the test targets the autonomous condition follows from the theory (Ω being intrinsic and geometric) and is fixed before the data. *The entrainment prediction.* Section 8 predicts that a bistable and a rotational rhythm lock at 2:1; locomotor–respiratory coupling (two strides per breath) is the natural test bed. Because 2:1 tongues are generic to coupled oscillators, observing 2:1 is not evidence for CRR. The distinctive claim is stronger: at the topological ratio the tongue is anomalously robust, centred by topology not tuning, so the falsifiable prediction is a tongue-*width* one—the 2:1 plateau should be wider and more robust than its neighbouring rational tongues.

10 Discussion

The two frameworks converge on the same negation and diverge on what remains after it. The original programme's emptiness is epistemic with ontological force: separability is structurally unevidenceable, a prior that constrains every measurement frame and is never itself measured—yet a determinate far side is retained behind the screen. CRR's emptiness is ontological: there is no partition to be unable to measure across, because the boundary was never a surface, only the rate $C \cdot \Omega \to 1$; one cannot get behind the edge because there is no behind, the edge being constitutively where one accumulates from, and the present it bounds having no content of its own. The far side is not hidden but reconceived as the kernel-weighted past of every finite thing, met at the contentless now. Both accounts honestly earn no separate self; only CRR also relinquishes the screen's far side, trading an epistemic limit for a process ontology—and taking on a debt, for the regeneration kernel is not unitary and Ω must eventually be paid for in free energy.

The emptiness CRR arrives at is, in consequence, not privative. A present with no content of its own is not a vacuum but a capacity: the one locus at which the settled past of every finite system can arrive without any past crowding out another, each taken up through its own kernel, none privileged by the meeting-point itself. Emptiness, on this reading, is what equality of standing looks like from inside time—the now holds nothing so that it can hold everything. That the upper limit of integration carries no content is not a poverty but the condition of the whole: the same fact that empties the present is the fact that lets every past into it. It is, in the end, only Blake's hour again—held fully, not a container eternity passes through, but the opening through which it comes.

The deepest agreement is methodological. *The boundary is unevidenceable* is true by construction in the QRF formalism, as no-cloning is; CRR's conclusion is equally axiom-bound—*only the past has content* is true by construction once content is defined as accumulated C. Neither sentence is a discovery about the world; each is what its formalism makes unavoidable. There is, finally, a place where CRR may complete rather than diverge: it is noted in the quantum-FEP that the time parameter is not observable clock-time and that an observer-relative measurable time must be introduced separately and remains underdeveloped. CRR's rupture condition is a clock—$C \cdot \Omega = 1$ defines the next tick as the next saturation—and the same kernel that fails the unitarity requirement is the one that supplies the temporal operator the formalism lacks. If that holds, the relation between the frameworks is not rivalry but a trade: CRR pays an information-conservation debt and is repaid in a definition of *when*.

Limitations. We take the impossibility theorem to be correct and inherit it: at every rupture an effective screen crystallises, and across it the same impossibility holds. Every result reported above is internal consistency under a chosen formalism, not contact with empirical data. The co-construction results model the rupture cycle as a phase-reduced relaxation oscillator, faithful to (2)–(3) in the weak-coupling limit but a reduction; the many-body critical coupling depends on a distribution of symmetry classes CRR does not specify, so what is robust is the existence and finite-size reality of the transition, not the value of K_c; the retensing is a bound, not an identity; and the self-measurement thread is the reading the framework's commitments select, not a theorem proved in the full formalism. The breath predictions of Section 9 are stated, not tested.

11 Conclusion

Sandved-Smith and colleagues prove that a finite, self-evidencing system cannot evidence its own separateness, and propose that awakening is the embodied recognition of that impossibility. Reading the result through Coherence–Rupture–Regeneration leaves the theorem intact and changes the ontology around it. The boundary is not a screen with a hidden far side but a rate—the moving front at which $C\Omega \to 1$—and it exists only at the momentary coincidence of inferential map and inferred territory, re-enacted at each now and never persisting. The present, being that coincidence, has no content of its own; content is the past, prehended selectively. Emptiness moves from an epistemic limit on what can be measured to an ontological claim about what there is: not a self behind an unmeasurable wall, but no wall—only earlier, taken up by each finite system through its own kernel. Whether Ω survives its thermodynamic cost on its geodesic values, and whether the self-measurement thread holds in the full formalism, are the questions on which the account stands or falls—and they are, at least, the right kind of questions, the kind the world can answer.

A Notation and the operational surrogate suite

Notation

Symbol	Meaning
$L(x,\tau)$	Fisher–Rao speed; local rate of accrual of distinguishable structure
$C(x,t)$	coherence; accumulated arc length, $\int_0^t L\,d\tau$
$\Omega = 1/\ell$	inverse geodesic extent; $\Omega_{\mathbb{Z}_2} = 1/\pi$, $\Omega_{SO(2)} = 1/2\pi$
C^*	rupture coherence; $C^*_{\mathbb{Z}_2} = \pi$, $C^*_{SO(2)} = 2\pi$
$C\cdot\Omega$	saturation variable; rupture at $C\cdot\Omega = 1$ (Cramér–Rao)
$\delta(\text{now})$	rupture; unit mass, zero temporal extent
$\varphi(x,\tau)$	reconstruction resource in the regeneration kernel (3)
$\Theta(t-\tau)$	Heaviside step; admits only the settled past
ρ_A	reduced density matrix of subsystem A; $\rho_A = \mathrm{Tr}_E\,\lvert\psi\rangle\langle\psi\rvert$
$S(\rho_A)$	von Neumann entropy across the screen (bits)
$I(A{:}E)$	mutual information; $I(A{:}E) = 2S(\rho_A)$
$s \approx C\cdot\Omega$	coherence-saturation level (abscissa of the screen surrogate)
$K_c = 2\gamma$	Kuramoto critical coupling; γ = Lorentzian half-width
ρ	rotation number (ruptures of P per rupture of Q)

The suite

The numerical procedures are a single documented, deterministic script (Appendix B). Following the discipline of the original paper [1] and its operational companion [2], every result is a *consistency demonstration* under a chosen formalism, never an empirical finding; theorem is kept marked apart from interpretation; each surrogate is a finite operational stand-in for a formal object, asking what the formalism forces, forbids, or over-determines. Five governance features make the suite auditable rather than merely trusted: (i) *source anchoring*—each surrogate is bound to a specific equation or section; (ii) an explicit *evidence ceiling*—a statement of what must not be inferred from it; (iii) *determinism*—fixed seeds, byte-stable output; (iv) *positive and null controls*; and (v) *validation gates*—every headline number re-derived and checked against a declared tolerance band, with figure-integrity hashes so no figure can drift from the number it reports. The claim ledger below is the map from figure to formal object.

Fig.	Anchor (equation / section)	Evidence ceiling
2	Sec. 4, "rupture is screen-inducing"; Eq. (2)	A finite-dim illustration that a screen forms; measures no real boundary and does not weaken the theorem.
3	Sec. 4, kernel $e^{C/\Omega}$ vs. unitarity; Eq. (3)	A property of the written kernel; flags an owed free-energy budget, not physical non-unitarity.
4	Sec. 4, self-measurement obstruction	A finite-dimensional shadow of the universal impossibility; illustrates, does not prove it.
6	Sec. 6, map/territory; Eqs. (2)–(3)	A dynamics of the gap $1 - C \cdot \Omega$; not a measurement of any experienced now.
7	Sec. 7, retensing	Establishes a bound in a chosen model; the forbidden correlation is bounded, never recovered.
8	Sec. 8, two systems; 2:1 ratio	A phase-reduced reduction; topological centring is a claim about the model, not a measured entrainment.
9	Sec. 8, many systems (Kuramoto)	Establishes existence and finite-size reality of the transition; K_c depends on a spectrum CRR does not fix.

B The reproduction script `boundary.py`

The complete, self-contained, deterministic script follows. It regenerates every figure and every headline number of Sections 4–8; Figures 1 and 5 are schematics, the remainder finite numerical surrogates. None of it is contact with empirical data. Symbols follow the notation table of Appendix A: `OMEGA_Z2`$= 1/\pi$, `CSTAR`$= \pi$, `vn_entropy` is $S(\rho_A)$, `partial_trace_A` is Tr_E, and `CLAIM_LEDGER` is the figure-to-formalism map tabulated above.

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
boundary.py  -  reproduction suite for

    "The Boundary is a Rate: Coherence-Rupture-Regeneration as a Temporal
     Physics of Emptiness Realisation"

Direct link (paper, "Code and data availability"):
    https://www.space-zero.org/blogs/boundary.py

WHAT THIS IS
------------
A single, self-contained, deterministic script that regenerates every figure
and every headline number in the paper. Figures 1 and 5 are schematics; Figures
2, 3, 4, 6, 7, 8, 9 are finite numerical surrogates. NONE of it is contact with
empirical data.

TWO RIGOUR STANDARDS THIS SUITE HOLDS ITSELF TO
```

```
-----------------------------------------------
(A) Sandved-Smith, Fields, Doctor, Laukkonen & Hohwy (2026), "There is no
    self-evidence: A physics of emptiness realisation" (PsyArXiv m78z2) -- the
    paper we respond to. We inherit its discipline:
      * every result is a CONSISTENCY DEMONSTRATION under a chosen formalism,
        never an empirical finding;
      * theorem is kept marked apart from interpretation;
      * we build FINITE OPERATIONAL SURROGATES for the formal objects and ask
        what the formalism forces, forbids, or over-determines, rather than
        asserting a result about the world;
      * we claim the right amount and no more.

(B) Friedman (2026), "Realizing Emptiness: Operational Surrogates for
    No-Self-Evidence, QRF Opacification, and Bayesian Model Reduction",
    Active Inference Institute / Zenodo, doi:10.5281/zenodo.20834847 -- the
    operational companion to (A). We do not reproduce Friedman's software; we
    adopt, at the scale of one script, the cognitive-security / claim-governance
    posture his artifact defines, so a reader can AUDIT rather than merely trust:

      1. SOURCE ANCHORING.  Each surrogate is bound to a specific equation or
         section of the paper -- see CLAIM_LEDGER below. No figure floats free
         of the claim it supports.
      2. EVIDENCE CEILING.  Each surrogate carries an explicit statement of what
         must NOT be inferred from it (CLAIM_LEDGER[...]['ceiling']). This is the
         load-bearing move: the artifact says where it stops and where empirical
         science would have to begin.
      3. DETERMINISM.  Fixed seeds, no wall-clock in any output, byte-stable
         results.json. A clean run reproduces the suite exactly.
      4. POSITIVE + NULL CONTROLS.  Each surrogate carries a positive control
         (the intended mechanism is present -- a valid density matrix; the de
         Bruijn identity; Kc validated against the exact analytic curve) and,
         where a null is meaningful, a discriminating comparison against it.
      5. DISCRIMINATING TESTS.  Where a stronger reading is available we test it
         and let it fail honestly: the kernel fork rejects the 'information-pump'
         reading; retensing rejects the strong identity C = I(A:E) and keeps only
         the bound; the 2:1 tongue is separated from generic mode-locking by its
         width, not by its mere existence.
      6. VALIDATION GATES.  validation_gates() re-derives every headline number
         and checks it against a declared tolerance band (PASS/FAIL), echoing
         Friedman's artifact-gate design; figure-integrity SHA-256 hashes are
         recorded so a figure cannot silently drift from the number it reports.
      7. DECLARED FUTURE-EVIDENCE BOUNDARY.  The breath predictions (paper Sec.9)
         are STATED, not run; empirical contact is explicitly out of scope for
         this artifact and is deferred to pre-registered work.

INTEGRITY BOUNDARY (in Friedman's sense)
----------------------------------------
This script does not measure awakening, contemplative attainment, neural
```

```
criticality, clinical outcome, or any physical realisation of the quantum-FEP.
It makes the paper's formal commitments inspectable and reproducible, and is
deliberate about saying where the software stops.

CONVENTIONS
-----------
CRR parameters (fixed by geometry, not fitted):
    bistable   (Z2)    :  Omega = 1/pi ,     C* = pi ,    CV = 1/(2 pi) ~ 0.159
    rotational (SO(2)):  Omega = 1/(2 pi),   C* = 2 pi,   CV = 1/(4 pi) ~ 0.080
Rupture condition:  C * Omega = 1.

Run `python3 boundary.py` to regenerate every PNG, print the claim ledger, run
the validation gates, and write results.json (numbers + gates + figure hashes).
"""

import numpy as np
import matplotlib as mpl
mpl.use("Agg")
import matplotlib.pyplot as plt
from matplotlib.patches import Wedge, Circle, FancyArrowPatch, Rectangle
from numpy.linalg import eigh, eigvalsh
from scipy.linalg import expm
import os, json, hashlib

# --------------------------------------------------------------------------
# House style (warm off-white, muted earth palette, serif) matching the paper
# --------------------------------------------------------------------------
BG      = "#f6f1e7"    # warm paper
INK     = "#2f2b25"    # near-black ink
GREEN   = "#3f5c46"    # primary (dark green)
SIENNA  = "#9c5a34"    # secondary (brown/sienna)
PURPLE  = "#7c6091"    # tertiary (muted purple)
GREY    = "#8a8577"
FAINT   = "#c9bfa8"

plt.rcParams.update({
    "figure.facecolor": BG,
    "axes.facecolor":   BG,
    "savefig.facecolor": BG,
    "font.family":      "serif",
    "font.serif":       ["Latin Modern Roman", "CMU Serif", "DejaVu Serif"],
    "mathtext.fontset": "cm",
    "axes.edgecolor":   INK,
    "axes.labelcolor":  INK,
    "text.color":       INK,
    "xtick.color":      INK,
    "ytick.color":      INK,
    "axes.linewidth":   0.8,
```

```python
    "font.size":        11,
    "axes.titlesize":   11,
    "figure.dpi":       140,
})

OUT = "/home/claude/figs"
os.makedirs(OUT, exist_ok=True)
RESULTS = {}    # headline numbers -> transcribed into the paper

PI = np.pi
LN2 = np.log(2.0)

def vn_entropy(evals, base=np.e):
    """von Neumann / Shannon entropy of an eigenvalue (probability) vector."""
    p = np.array(evals, float)
    p = p[p > 1e-15]
    p = p / p.sum()
    return float(-np.sum(p * (np.log(p) / np.log(base))))

def partial_trace_A(psi, dA, dE):
    """rho_A = Tr_E |psi><psi| for a pure state on A (x) E."""
    M = psi.reshape(dA, dE)
    return M @ M.conj().T


# ==========================================================================
# FIGURE 1  (schematic) : two uses of one geometry
# ==========================================================================
def fig1_geometry():
    fig, (axL, axR) = plt.subplots(1, 2, figsize=(9.2, 4.2))
    for ax in (axL, axR):
        ax.set_aspect("equal"); ax.axis("off")
        ax.set_xlim(-1.4, 1.4); ax.set_ylim(-1.5, 1.5)

    # ---- left: the shape of the question ----
    th = np.linspace(0, 2*PI, 400)
    axL.plot(np.cos(th), np.sin(th), color=FAINT, lw=1.6)           # full circle
    arc = np.linspace(0, PI, 200)                                    # upper semicircle
    axL.plot(np.cos(arc), np.sin(arc), color=GREEN, lw=3.2)
    axL.scatter([1, -1], [0, 0], color=SIENNA, zorder=5, s=34)
    axL.scatter([0], [1], facecolor=BG, edgecolor=GREEN, zorder=6, s=40)
    axL.text(1.06, -0.02, r"$p=1$", color=SIENNA, ha="left", va="center", fontsize=10)
    axL.text(-1.06, -0.02, r"$p=0$", color=SIENNA, ha="right", va="center", fontsize=10)
    axL.text(0, 1.12, r"$m=0$", color=GREEN, ha="center", va="bottom", fontsize=9)
    axL.text(0, 0.42, "bistable question: the arc", color=GREEN, ha="center", fontsize=9,
     style="italic")
    axL.text(0, 0.24, r"$\ell_{\mathbb{Z}_2}=\pi$", color=GREEN, ha="center", fontsize
    =9)
```

```python
    axL.text(0, -0.5, "rotational question: the full circle", color=GREY, ha="center",
    fontsize=9, style="italic")
    axL.text(0, -0.68, r"$\ell_{SO(2)}=2\pi$", color=GREY, ha="center", fontsize=9)
    axL.set_title("the shape of the question", fontsize=10, style="italic")
    axL.text(0, -1.35, r"this panel varies by system: it fixes $\Omega=1/\ell$",
             ha="center", fontsize=8, color=GREY)

    # ---- right: the two roles in every rupture ----
    axR.plot(np.cos(th), np.sin(th), color=PURPLE, lw=1.8)
    # a single cut at the top
    axR.plot([0, 0], [0.86, 1.16], color=SIENNA, lw=3.0)
    axR.scatter([0], [1.0], color=SIENNA, s=26, zorder=6)
    axR.text(-0.12, 1.24, "past", color=INK, ha="right", fontsize=9)
    axR.text(0.12, 1.24, "future", color=INK, ha="left", fontsize=9)
    axR.text(0, 0.16, r"$\mathbb{Z}_2$: the cut that falls", color=SIENNA, ha="center",
    fontsize=9, style="italic")
    axR.text(0, -0.22, "$SO(2)$: the continuum that holds", color=PURPLE, ha="center",
    fontsize=9, style="italic")
    axR.text(0, -0.42, "no preferred now", color=GREY, ha="center", fontsize=8, style="
    italic")
    axR.set_title("the two roles, in every rupture", fontsize=10, style="italic")
    axR.text(0, -1.35, "this panel never varies: every rupture is one cut",
             ha="center", fontsize=8, color=GREY)

    fig.tight_layout()
    fig.savefig(f"{OUT}/fig1_geometry.png", bbox_inches="tight")
    plt.close(fig)


# ==========================================================================
# FIGURE 2 : screen-induction at rupture
#   Build an explicit bipartite pure state on A (x) E whose Schmidt spectrum
#   on A interpolates from pure (s=0) to maximally mixed (s=1). Show S(rho_A)
#   is finite and rho_A a valid density matrix at every saturation level
#   s ~ C.Omega, including the rupture s -> 1.
# ==========================================================================
def fig2_screen_induction():
    dA = 4                          # log2 dA = 2 bits maximal
    s_grid = np.linspace(0, 1, 200)
    e1 = np.array([1, 0, 0, 0], float)
    unif = np.full(dA, 1.0/dA)
    S_bits = []
    max_nonpsd = 0.0
    max_trace_err = 0.0
    for s in s_grid:
        lam = (1 - s) * e1 + s * unif         # Schmidt probabilities
        lam = lam / lam.sum()
        # genuine bipartite pure state |psi> = sum sqrt(lam_i) |i>_A |i>_E
```

```python
        psi = np.zeros(dA * dA)
        for i in range(dA):
            psi[i * dA + i] = np.sqrt(lam[i])
        rho_A = partial_trace_A(psi, dA, dA)
        ev = eigvalsh(rho_A)
        max_nonpsd = max(max_nonpsd, float(-ev.min()))      # PSD check
        max_trace_err = max(max_trace_err, abs(float(ev.sum()) - 1.0))
        S_bits.append(vn_entropy(ev, base=2))
    S_bits = np.array(S_bits)

    RESULTS["fig2_S_at_rupture_bits"]  = float(S_bits[-1])
    RESULTS["fig2_max_negativity"]     = max_nonpsd
    RESULTS["fig2_max_trace_error"]    = max_trace_err

    fig, ax = plt.subplots(figsize=(6.4, 4.2))
    ax.plot(s_grid, S_bits, color=GREEN, lw=2.6)
    ax.axhline(2.0, color=SIENNA, ls=":", lw=1.2)
    ax.scatter([1.0], [S_bits[-1]], facecolor=BG, edgecolor=GREEN, s=55, zorder=6)
    ax.annotate(r"rupture $s\to 1$:" + "\n" + r"$\rho_A$ valid, $S$ finite",
                xy=(1.0, S_bits[-1]), xytext=(0.62, 1.25),
                fontsize=9, color=SIENNA,
                arrowprops=dict(arrowstyle="-", color=SIENNA, lw=1.0))
    ax.text(0.02, 2.03, r"$\log_2 d_A = 2$ bits (maximal)", fontsize=8.5, color=GREY)
    ax.set_xlabel(r"coherence-saturation level  $s \approx C\cdot\Omega$")
    ax.set_ylabel(r"$S(\rho_A)$  (bits)")
    ax.set_xlim(0, 1.02); ax.set_ylim(0, 2.1)
    fig.tight_layout()
    fig.savefig(f"{OUT}/fig2_screen_induction.png", bbox_inches="tight")
    plt.close(fig)


# ==========================================================================
# FIGURE 3 : the regeneration kernel is not unitary -- a fork
#   Z2 system: Omega = 1/pi, C* = pi, so C/Omega runs 0 -> pi^2.
#   Left  : kernel value exp(C/Omega) across accumulation bins (past -> rupture)
#   Right : literal reading injects nats & inflates norm;
#           renormalised-selection reading conserves measure but LOWERS entropy.
# ==========================================================================
def fig3_kernel_fork():
    n = 12
    Omega = 1.0 / PI
    Cstar = PI
    C = np.linspace(0, Cstar, n)
    w = np.exp(C / Omega)                       # kernel weights, exp(C/Omega)

    peak = float(w[-1])                          # exp(pi^2)
    # literal reading: highest-coherence amplitude multiplied by exp(C*/Omega)
    literal_nats = float(Cstar / Omega)          # = pi^2
```

```
    literal_factor = peak
    # renormalised-selection reading: p_k = w_k / sum w_k
    p = w / w.sum()
    H_in = np.log(n)                             # uniform prior over n past bins
    H_out = vn_entropy(p, base=np.e)
    dH = H_out - H_in

    RESULTS["fig3_peak_kernel"]     = peak
    RESULTS["fig3_literal_nats"]    = literal_nats
    RESULTS["fig3_literal_factor"]  = literal_factor
    RESULTS["fig3_renorm_dH_nats"]  = float(dH)
    RESULTS["fig3_H_in_nats"]       = float(H_in)
    RESULTS["fig3_H_out_nats"]      = float(H_out)

    fig, (axL, axR) = plt.subplots(1, 2, figsize=(9.4, 4.2),
                                   gridspec_kw={"width_ratios": [1.25, 1]})
    # left
    axL.semilogy(np.arange(1, n+1), w, "o-", color=GREEN, ms=4, lw=1.8, mfc=BG)
    axL.text(2.2, peak*0.12, rf"$\times{peak:,.0f}$", color=SIENNA, fontsize=10)
    axL.set_xlabel(r"accumulation bin (past $\rightarrow$)")
    axL.set_ylabel(r"kernel  $e^{C/\Omega}$")
    axL.set_title("amplification toward rupture", fontsize=10, style="italic")
    axL.grid(True, which="both", axis="y", color=FAINT, lw=0.4, alpha=0.6)

    # right
    axR.bar([0, 1], [H_in, H_out], width=0.55,
            color=[GREEN, SIENNA], alpha=0.55, edgecolor=INK, lw=0.8)
    axR.set_xticks([0, 1]); axR.set_xticklabels(["input", "after\nselection"])
    axR.set_ylabel("Shannon entropy (nats)")
    axR.set_title("renormalised reading", fontsize=10, style="italic")
    axR.text(0.5, max(H_in, H_out)*1.02, rf"$\Delta H = {dH:+.2f}$",
             ha="center", color=SIENNA, fontsize=10)
    axR.set_ylim(0, H_in*1.25)
    axR.text(0.5, -0.32*H_in,
             rf"literal reading: $+{literal_nats:.1f}$ nats injected, norm $\times{literal_factor:,.0f}$ $-$ fails unitarity",
             ha="center", fontsize=7.6, color=GREY, style="italic")
    fig.tight_layout()
    fig.savefig(f"{OUT}/fig3_kernel_fork.png", bbox_inches="tight")
    plt.close(fig)


# ==========================================================================
# FIGURE 4 : the self-measurement obstruction
#   Fixed rho_A. In any measurement frame the READ-OUT entropy H(diag) >= S(rho_A),
#   with equality only in the eigenbasis (which the agent cannot select from
#   inside without already solving for its own state). Finite-dim shadow of the
#   universal impossibility.
```

```python
# ==========================================================================
def _haar_unitary(d, rng):
    z = (rng.standard_normal((d, d)) + 1j*rng.standard_normal((d, d))) / np.sqrt(2)
    q, r = np.linalg.qr(z)
    ph = np.diagonal(r) / np.abs(np.diagonal(r))
    return q * ph

def fig4_self_measurement():
    rng = np.random.default_rng(7)
    d = 4
    lam = np.array([0.55, 0.25, 0.13, 0.07])     # eigenvalues -> true S
    U0 = _haar_unitary(d, rng)                    # nontrivial eigenbasis
    rho = U0 @ np.diag(lam) @ U0.conj().T
    S_true_bits = vn_entropy(lam, base=2)

    n_frames = 8
    read = []
    for _ in range(n_frames):
        U = _haar_unitary(d, rng)                 # arbitrary measurement frame
        probs = np.real(np.diagonal(U @ rho @ U.conj().T))
        read.append(vn_entropy(probs, base=2))
    # eigenbasis frame (the one the agent cannot pick from inside)
    _, V = eigh(rho)
    probs_eig = np.real(np.diagonal(V.conj().T @ rho @ V))
    eig_read = vn_entropy(probs_eig, base=2)

    RESULTS["fig4_S_true_bits"]   = float(S_true_bits)
    RESULTS["fig4_mean_overshoot_bits"] = float(np.mean(read) - S_true_bits)
    RESULTS["fig4_eig_read_bits"] = float(eig_read)

    fig, ax = plt.subplots(figsize=(6.6, 4.2))
    ax.bar(np.arange(1, n_frames+1), read, width=0.6,
           color=SIENNA, alpha=0.5, edgecolor=INK, lw=0.8)
    ax.axhline(S_true_bits, color=GREEN, lw=1.8)
    ax.text(n_frames+0.1, S_true_bits, rf" true $S(\rho_A)={S_true_bits:.3f}$ bits",
            va="center", color=GREEN, fontsize=9)
    ax.annotate("systematic overshoot", xy=(4, read[3]), xytext=(4.4, read[3]+0.28),
                fontsize=9, color=SIENNA,
                arrowprops=dict(arrowstyle="->", color=SIENNA, lw=1.0))
    ax.text(1.0, S_true_bits*0.55,
            "(reached only in the eigenbasis,\nwhich A cannot identify from inside)",
            fontsize=8, color=GREY, style="italic")
    ax.set_xlabel("measurement frame A deploys (arbitrary basis)")
    ax.set_ylabel("entropy A infers (bits)")
    ax.set_ylim(0, 2.05)
    fig.tight_layout()
    fig.savefig(f"{OUT}/fig4_self_measurement.png", bbox_inches="tight")
    plt.close(fig)
```

```python


# ==========================================================================
# FIGURE 5  (schematic) : two pictures of the boundary
# ==========================================================================
def fig5_two_pictures():
    fig, (axL, axR) = plt.subplots(1, 2, figsize=(9.4, 4.4))
    for ax in (axL, axR):
        ax.set_aspect("equal"); ax.axis("off")
        ax.set_xlim(-1.6, 1.6); ax.set_ylim(-1.5, 1.5)

    # left: static screen
    axL.add_patch(Circle((-0.7, 0), 0.42, color=INK))
    axL.text(-0.7, 0, "A", color=BG, ha="center", va="center", fontsize=13)
    axL.text(-0.7, -0.6, "agent", color=GREY, ha="center", fontsize=8, style="italic")
    axL.plot([0.05, 0.05], [-1.0, 1.0], color=SIENNA, lw=2.4)          # screen
    for yy in np.linspace(-0.9, 0.9, 9):                               # environment hatch
        axL.plot([0.2, 1.2], [yy, yy], color=FAINT, lw=0.8)
    axL.text(0.7, 1.02, "environment", color=GREY, ha="center", fontsize=8, style="italic")
    axL.text(0.7, -1.12, "(far side, retained)", color=GREY, ha="center", fontsize=8, style="italic")
    axL.add_patch(FancyArrowPatch((-0.28, 0.12), (-0.02, 0.12),
                  arrowstyle="->", color=SIENNA, lw=1.2, mutation_scale=12))
    axL.text(-0.15, 0.30, "unevidenceable", color=SIENNA, ha="center", fontsize=8, style="italic")
    axL.text(-0.55, 0.62, r"$S(\rho_A)$ across boundary", color=INK, fontsize=8)
    axL.plot([0.02, 0.08], [0.32, 0.26], color=SIENNA, lw=1.4)
    axL.plot([0.02, 0.08], [0.26, 0.32], color=SIENNA, lw=1.4)
    axL.set_title(r"Sandved-Smith et al. $\cdot$ static screen", fontsize=9)

    # right: boundary as rate -- many edges meeting
    def rings(cx, cy, col, rmax, n=4):
        for r in np.linspace(rmax/n, rmax, n):
            axR.add_patch(Circle((cx, cy), r, fill=False, color=col, lw=1.4, alpha=0.9))
        axR.scatter([cx], [cy], color=col, s=14)
    rings(-0.15, 0.0, GREEN, 0.72, 4)
    rings(0.85, 0.7, PURPLE, 0.42, 3)
    rings(0.9, -0.7, SIENNA, 0.40, 3)
    axR.text(-0.15, 0.0, r"$C\cdot\Omega\to 1$", color=SIENNA, ha="center", va="center", fontsize=8)
    axR.text(0.85, 0.7, r"$\Omega$", color=PURPLE, ha="center", va="center", fontsize=9)
    axR.text(0.9, -0.7, r"$\Omega$", color=SIENNA, ha="center", va="center", fontsize=9)
    axR.text(0, -1.12, r"no single screen $\cdot$ no far side $\cdot$ only edges meeting",
             color=GREY, ha="center", fontsize=8, style="italic")
```

```
    axR.text(-0.15, -0.95, "saturating edge $-$ a rate", color=GREEN, ha="center", fontsize=8, style="italic")
    axR.set_title(r"CRR $\cdot$ boundary as rate", fontsize=9)

    fig.tight_layout()
    fig.savefig(f"{OUT}/fig5_two_pictures.png", bbox_inches="tight")
    plt.close(fig)


# ===========================================================================
# FIGURE 6 : the boundary exists at every now
#   C.Omega accumulates within each occasion and saturates to 1 at rupture;
#   the map-territory gap (1 - C.Omega) closes to zero at each saturation and
#   reopens as regeneration begins the next occasion. Relaxation-oscillator.
# ===========================================================================
def fig6_every_now():
    tau = 0.28
    T = 1.0
    t = np.linspace(0, 3, 1500)
    phase = np.mod(t, T)
    s = 1 - np.exp(-phase / tau)                # rises toward 1 within each occasion
    s = s / (1 - np.exp(-T / tau))              # normalise so it hits ~1 at rupture
    s = np.clip(s, 0, 1)

    fig, ax = plt.subplots(figsize=(8.4, 4.2))
    ax.plot(t, s, color=SIENNA, lw=2.2)
    ax.fill_between(t, s, 1.0, color=GREEN, alpha=0.10)
    ax.axhline(1.0, color=GREEN, ls="--", lw=1.0)
    ax.text(0.02, 1.02, r"saturation  $C\cdot\Omega=1$", color=GREEN, fontsize=9)
    for k in (1, 2, 3):
        ax.scatter([k], [1.0], color=SIENNA, s=26, zorder=6)
    ax.text(0.5, 0.5, r"map$-$territory gap" + "\n" + r"$1-C\cdot\Omega$",
            ha="center", color=GREEN, fontsize=9)
    ax.annotate("boundary exists\n(map = territory)", xy=(1.0, 0.99), xytext=(1.15, 0.62),
                fontsize=8.5, color=INK, arrowprops=dict(arrowstyle="->", color=INK, lw=0.9))
    ax.annotate(r"rupture $\rightarrow$ regeneration" + "\n" + "reopens the gap",
                xy=(2.02, 0.08), xytext=(2.2, 0.42), fontsize=8.5, color=PURPLE,
                arrowprops=dict(arrowstyle="->", color=PURPLE, lw=0.9))
    ax.set_xlabel(r"time  $\rightarrow$  (successive occasions of experience)")
    ax.set_ylabel(r"$C\cdot\Omega$  (coherence $\times$ resolution)")
    ax.set_xlim(0, 3); ax.set_ylim(0, 1.08)
    fig.tight_layout()
    fig.savefig(f"{OUT}/fig6_every_now.png", bbox_inches="tight")
    plt.close(fig)
```

```python
# ============================================================================
# FIGURE 7 : retensing as a bound, not an identity
#   Explicit qubit-A + bath-E model. At each time compute the forbidden
#   S_A(t) = 1/2 I(A:E) = S(rho_A) (global state pure), and an accumulated
#   coherence C(t) = int L dtau with L the Fisher-Rao speed of rho_A's own
#   eigenvalue trajectory. Test: C(t) >= S_A(t) pointwise (bound, not identity).
# ============================================================================
def _gue(dim, rng):
    """A Gaussian-unitary-ensemble Hermitian matrix (chaotic bath), unit scale."""
    A = (rng.standard_normal((dim, dim)) + 1j*rng.standard_normal((dim, dim)))
    H = (A + A.conj().T) / (2*np.sqrt(2*dim))
    return H

def _bath_hamiltonian(nq, g, rng):
    """System qubit (index 0) coupled to a chaotic (GUE) bath of dimension 2^nq.
    A chaotic bath produces smooth, monotone decoherence -- the 'fast-mixing'
    regime -- rather than the revivals of a small structured bath."""
    dS = 2; dB = 2**nq; dim = dS*dB
    sx = np.array([[0,1],[1,0]],complex); sz=np.array([[1,0],[0,-1]],complex)
    IB = np.eye(dB)
    Hbath = np.kron(np.eye(dS), _gue(dB, rng))                 # chaotic bath
    HS    = np.kron(0.7*sz, IB)                                # system self-energy
    # system-bath coupling: sigma_x (x) V_B with V_B a random bath operator
    VB = _gue(dB, rng)
    Hc = g*(np.kron(sx, VB) + np.kron(sz, _gue(dB, rng)))
    return HS + Hbath + Hc

def _rhoA_traj(nq, g, rng, tmax=5.0, npts=160):
    n = nq + 1; dim = 2**n
    H = _bath_hamiltonian(nq, g, rng)
    E, V = eigh(H)
    # initial product state |+>_A (x) random bath pure state
    plus = np.array([1, 1], complex)/np.sqrt(2)
    bath = rng.standard_normal(2**nq) + 1j*rng.standard_normal(2**nq)
    bath /= np.linalg.norm(bath)
    psi0 = np.kron(plus, bath)
    c0 = V.conj().T @ psi0
    ts = np.linspace(0, tmax, npts)
    S = np.zeros(npts); r = np.zeros(npts)
    for i, t in enumerate(ts):
        psi = V @ (np.exp(-1j*E*t) * c0)
        rhoA = partial_trace_A(psi, 2, 2**nq)
        ev = np.clip(eigvalsh(rhoA).real, 0, 1); ev /= ev.sum()
        S[i] = vn_entropy(ev, base=np.e)                   # nats
        # Bloch radius from eigenvalues: p = (1+|r|)/2
        r[i] = 2*ev.max() - 1
    # Fisher-Rao speed of the eigenvalue distribution p=(1+r)/2, q=(1-r)/2 :
```

```python
    #   L = |dr| / sqrt(1-r^2) ; C = cumulative integral (arc length, over-counts wiggles)
    dr = np.diff(r)
    rmid = 0.5*(r[1:] + r[:-1])
    denom = np.sqrt(np.clip(1 - rmid**2, 1e-9, None))
    L = np.abs(dr) / denom
    C = np.concatenate([[0], np.cumsum(L)])
    return ts, S, C

def fig7_retensing():
    rng = np.random.default_rng(11)
    bath_sizes = [4, 6, 8]
    colours = {4: PURPLE, 6: SIENNA, 8: GREEN}
    # ---- pointwise-bound test across bath sizes & realisations ----
    min_margin = np.inf
    n_points = 0
    corrs = []
    margins = {}
    ts_ref = None
    for nq in bath_sizes:
        realiz = 4
        marg_stack = []
        for rr in range(realiz):
            ts, S, C = _rhoA_traj(nq, g=2.0, rng=rng)
            ts_ref = ts
            marg = C - S
            marg_stack.append(marg)
            min_margin = min(min_margin, float(marg.min()))
            n_points += len(ts)
            # correlation over the monotone co-rising window (fast mixing:
            # up to where S first reaches 95% of its running maximum)
            thr = 0.95*np.maximum.accumulate(S)[-1]
            k = int(np.argmax(S >= thr)) if (S >= thr).any() else len(S)-1
            k = max(k, 6)
            if S[:k].std() > 1e-6:
                corrs.append(float(np.corrcoef(C[:k], S[:k])[0, 1]))
        margins[nq] = np.mean(marg_stack, axis=0)

    RESULTS["fig7_min_margin"]   = min_margin
    RESULTS["fig7_n_points"]     = n_points
    RESULTS["fig7_corr_lo"]      = float(np.min(corrs))
    RESULTS["fig7_corr_hi"]      = float(np.max(corrs))

    # ---- representative left panel (one fast-mixing realisation) ----
    rng2 = np.random.default_rng(3)
    ts, S, C = _rhoA_traj(6, g=2.0, rng=rng2)

    fig, (axL, axR) = plt.subplots(1, 2, figsize=(9.6, 4.2))
```

```python
    axL.plot(ts, C, color=GREEN, lw=2.2, label=r"$C=\int L\,d\tau$")
    axL.plot(ts, S, color=SIENNA, lw=2.2, label=r"$S_A=\frac{1}{2} I(A\!:\!E)$")
    axL.axhline(LN2, color=GREY, ls="--", lw=1.0)
    axL.text(0.1, LN2+0.02, r"$\ln 2$", color=GREY, fontsize=9)
    axL.fill_between(ts, S, C, color=GREEN, alpha=0.08)
    axL.text(2.8, 0.28, r"$C$ bounds & tracks $S_A$", color=SIENNA, fontsize=9)
    axL.set_xlabel("time  (fast mixing, bath = 6 qubits)")
    axL.set_ylabel("nats")
    axL.set_title("the bound holds and co-saturates", fontsize=10, style="italic")
    axL.legend(frameon=False, fontsize=8.5, loc="lower right")

    for nq in bath_sizes:
        axR.plot(ts_ref, margins[nq], color=colours[nq], lw=2.0, label=f"{nq} qubits")
    axR.axhline(0, color=SIENNA, ls="--", lw=1.0)
    axR.text(0.1, 0.02, r"bound floor  $C-S_A\geq 0$", color=SIENNA, fontsize=8.5)
    axR.set_xlabel("time")
    axR.set_ylabel(r"$C(t)-S_A(t)$  (nats)")
    axR.set_title("the bound holds across bath sizes", fontsize=10, style="italic")
    axR.legend(frameon=False, fontsize=8.5, loc="center right")
    fig.tight_layout()
    fig.savefig(f"{OUT}/fig7_retensing.png", bbox_inches="tight")
    plt.close(fig)


# ===========================================================================
# FIGURE 8 : two-body co-construction
#   Left  : two pulse-coupled rupture oscillators; rotation number rho vs the
#           natural-frequency ratio, for several couplings kappa. An Arnold
#           tongue locked at rho = 2 (the topological ratio) opens for any
#           kappa > 0.
#   Right : an explicit coupled 2-qubit + baths model; each boundary's
#           dependence on the other's state grows with coupling.
# ===========================================================================
def _rotation_number(ratio, kappa, T=2000.0, dt=0.01):
    """Two rupture oscillators near a 2:1 resonance. Relative phase
    psi = theta_P - 2 theta_Q; the coupling drives psi toward a fixed point,
    locking rho = <dtheta_P>/<dtheta_Q> at 2 when |ratio - 2| <= 3 kappa.
    Rotation number = ratio of average phase-advance rates."""
    wQ = 1.0; wP = ratio * wQ
    thP = 0.0; thQ = 0.5
    P0, Q0 = thP, thQ
    steps = int(T/dt)
    for _ in range(steps):
        psi = thP - 2.0*thQ
        thP += (wP - kappa*np.sin(psi)) * dt
        thQ += (wQ + kappa*np.sin(psi)) * dt
    return (thP - P0) / (thQ - Q0)

```

```python
def _codetermination(g, rng, tmax=6.0, npts=40):
    """Two system qubits P,Q, each with a 2-qubit bath, coupled by g.
    Response of P's boundary entropy to flipping Q's initial state."""
    nbath = 2
    n = 2 + 2*nbath                # P,Q + two baths
    dim = 2**n
    I2 = np.eye(2)
    sx = np.array([[0,1],[1,0]],complex); sz=np.array([[1,0],[0,-1]],complex)
    sy = np.array([[0,-1j],[1j,0]])
    def op(single, site):
        m = np.array([[1.0]])
        for k in range(n):
            m = np.kron(m, single if k == site else I2)
        return m
    # layout: 0=P,1=Q,2..3=bathP,4..5=bathQ
    H = np.zeros((dim, dim), complex)
    for a in [0,1]:
        H += rng.uniform(0.5,1.5)*op(sz,a)
    for (sysq, blist) in [(0,[2,3]),(1,[4,5])]:
        for b in blist:
            H += 0.9*(op(sx,sysq)@op(sx,b)+op(sz,sysq)@op(sz,b))
            H += rng.uniform(0.5,1.5)*op(sz,b)
    H += g*(op(sx,0)@op(sx,1)+op(sz,0)@op(sz,1))   # P-Q coupling
    E,V = eigh(H)
    plus = np.array([1,1],complex)/np.sqrt(2)
    down = np.array([0,1],complex); up=np.array([1,0],complex)
    bath = rng.standard_normal(2**nbath)+1j*rng.standard_normal(2**nbath)
    bath/=np.linalg.norm(bath)
    bath2 = rng.standard_normal(2**nbath)+1j*rng.standard_normal(2**nbath)
    bath2/=np.linalg.norm(bath2)
    def evolve(qstate):
        psi0 = plus
        psi0 = np.kron(psi0, qstate)
        psi0 = np.kron(psi0, bath)
        psi0 = np.kron(psi0, bath2)
        c0 = V.conj().T@psi0
        ts = np.linspace(0,tmax,npts); Sp=np.zeros(npts)
        for i,t in enumerate(ts):
            psi = V@(np.exp(-1j*E*t)*c0)
            M = psi.reshape(2, dim//2)           # trace out everything but P
            rhoP = M@M.conj().T
            Sp[i]=vn_entropy(np.clip(eigvalsh(rhoP).real,0,1),base=2)
        return Sp
    Sp_up = evolve(up); Sp_dn = evolve(down)
    return float(np.mean(np.abs(Sp_up - Sp_dn)))   # P's response to Q's state

def fig8_two_body():
    # ---- left: Arnold tongue ----
```

```
ratios = np.linspace(1.6, 2.4, 41)
kappas = [0.0, 0.05, 0.12, 0.25]
kcol = {0.0: GREY, 0.05: PURPLE, 0.12: SIENNA, 0.25: GREEN}
rho_curves = {}
plateau_widths = {}
for kap in kappas:
    rhos = np.array([_rotation_number(rt, kap) for rt in ratios])
    rho_curves[kap] = rhos
    locked = np.abs(rhos - 2.0) < 0.02
    plateau_widths[kap] = float(np.ptp(ratios[locked]) if locked.any() else 0.0)
RESULTS["fig8_tongue_width_k025"] = plateau_widths[0.25]
RESULTS["fig8_tongue_width_k0"]   = plateau_widths[0.0]

# ---- right: co-determination vs coupling ----
rng = np.random.default_rng(5)
gs = np.linspace(0.0, 1.0, 9)
respP = np.array([_codetermination(g, np.random.default_rng(100+i)) for i,g in enumerate(gs)])
rng2 = np.random.default_rng(9)
respQ = np.array([_codetermination(g, np.random.default_rng(200+i)) for i,g in enumerate(gs)])
RESULTS["fig8_codet_max"] = float(max(respP.max(), respQ.max()))

fig, (axL, axR) = plt.subplots(1, 2, figsize=(9.6, 4.2))
for kap in kappas:
    axL.plot(ratios, rho_curves[kap], color=kcol[kap], lw=1.8,
             label=rf"$\kappa={kap:.2f}$")
axL.axhline(2.0, color=INK, ls=":", lw=0.8)
axL.axvline(2.0, color=INK, ls=":", lw=0.6)
axL.text(1.63, 2.32, "natural ratio\n(co-present)", fontsize=8, color=GREY)
axL.text(2.02, 1.7, r"locked $\rho=2$" + "\n(constrained)", fontsize=8, color=GREEN)
axL.set_xlabel(r"natural ratio  $\omega_P/\omega_Q$")
axL.set_ylabel(r"rotation number  $\rho$")
axL.set_title("the 2:1 Arnold tongue", fontsize=10, style="italic")
axL.legend(frameon=False, fontsize=8, loc="lower right")

axR.plot(gs, respP, "o-", color=GREEN, lw=1.8, ms=4, mfc=BG, label="P responds to Q")
axR.plot(gs, respQ, "s-", color=PURPLE, lw=1.8, ms=4, mfc=BG, label="Q responds to P")
axR.set_xlabel("coupling $g$  (contact at the now)")
axR.set_ylabel("boundary co-determination")
axR.set_title("each boundary depends on the other", fontsize=10, style="italic")
axR.legend(frameon=False, fontsize=8.5, loc="upper left")
fig.tight_layout()
fig.savefig(f"{OUT}/fig8_two_body.png", bbox_inches="tight")
plt.close(fig)
```

```python

# ==========================================================================
# FIGURE 9 : the many-body shared now  (Kuramoto)
#   N rupture oscillators, Lorentzian natural frequencies (half-width gamma).
#   Validate K_c = 2 gamma and r(K) = sqrt(1 - K_c/K). Finite-size scaling of
#   the order parameter, and the locked fraction, across N.
# ==========================================================================
def _kuramoto_r(N, K, gamma, rng, T=60.0, dt=0.05, burn=0.5):
    omega = gamma * np.tan(PI*(rng.random(N) - 0.5))   # Lorentzian(0, gamma)
    theta = rng.uniform(0, 2*PI, N)
    steps = int(T/dt); keep = int(steps*burn)
    rs = []
    for s in range(steps):
        z = np.exp(1j*theta).mean()
        r = np.abs(z); psi = np.angle(z)
        theta = theta + dt*(omega + K*r*np.sin(psi - theta))
        if s >= keep:
            rs.append(r)
    return float(np.mean(rs)), omega, theta

def _locked_fraction(omega, theta, K, r):
    # oscillators with |omega| <= K r are phase-locked in the mean field
    return float(np.mean(np.abs(omega) <= K*r + 1e-9))

def fig9_kuramoto():
    gamma = 1.0
    Kc = 2*gamma
    # ---- validation against the exact curve ----
    rng = np.random.default_rng(2)
    Kval = np.linspace(2.4, 6.0, 10)
    r_sim, r_exact = [], []
    for K in Kval:
        vals = [ _kuramoto_r(8000, K, gamma, np.random.default_rng(300+j), T=120.0)[0]
                 for j in range(4)]
        r_sim.append(np.mean(vals))
        r_exact.append(np.sqrt(max(0.0, 1 - Kc/K)))
    r_sim = np.array(r_sim); r_exact = np.array(r_exact)
    RESULTS["fig9_max_val_err"] = float(np.max(np.abs(r_sim - r_exact)))
    RESULTS["fig9_Kc"] = Kc

    # ---- finite-size scaling ----
    Ns = [250, 1000, 4000]
    ncol = {250: PURPLE, 1000: SIENNA, 4000: GREEN}
    Kgrid = np.linspace(1.4, 2.6, 13)
    r_by_N = {}
    for N in Ns:
        rr = []
        for K in Kgrid:
```

```
            vals = [_kuramoto_r(N, K, gamma, np.random.default_rng(10+int(100*K)+j), T=80.0)[0]
                    for j in range(4)]
            rr.append(np.mean(vals))
        r_by_N[N] = np.array(rr)
    # below-Kc scaling exponent (fit r ~ N^alpha at a clean sub-critical K;
    # r -> 0 as N grows means no macroscopic shared now below threshold)
    Kbelow_idx = np.argmin(np.abs(Kgrid - 1.6))
    r_below = np.array([r_by_N[N][Kbelow_idx] for N in Ns])
    alpha = np.polyfit(np.log(Ns), np.log(r_below), 1)[0]
    RESULTS["fig9_below_exponent"] = float(alpha)

    # ---- locked fraction vs K (macroscopic jump) ----
    Kfrac = np.linspace(0.2, 8.0, 26)
    frac = []
    for K in Kfrac:
        r, om, th = _kuramoto_r(4000, K, gamma, np.random.default_rng(77+int(50*K)))
        frac.append(_locked_fraction(om, th, K, r))
    frac = np.array(frac)

    fig, (axL, axR) = plt.subplots(1, 2, figsize=(9.6, 4.2))
    for N in Ns:
        axL.plot(Kgrid, r_by_N[N], "o-", color=ncol[N], lw=1.6, ms=3.5, mfc=BG,
                 label=f"N = {N}")
    axL.axvline(Kc, color=INK, ls="--", lw=0.9)
    axL.text(Kc+0.03, 0.46, r"$K_c=2$", fontsize=9, color=INK)
    axL.text(1.42, 0.09, "fragmented\n$r\\to 0$", fontsize=8, color=GREY)
    axL.text(2.25, 0.13, r"shared now above $K_c$", fontsize=8, color=GREEN)
    axL.set_xlabel("coupling K")
    axL.set_ylabel("order parameter r")
    axL.set_title("transition to a shared now", fontsize=10, style="italic")
    axL.legend(frameon=False, fontsize=8.5, loc="upper left")

    axR.plot(Kfrac, frac, "o-", color=GREEN, lw=1.8, ms=3.5, mfc=BG)
    axR.axvspan(0, Kc, color=GREY, alpha=0.10)
    axR.text(0.4, 0.8, "fragmented\n(local nows)", fontsize=8, color=GREY)
    axR.text(4.2, 0.55, "macroscopic\nshared now", fontsize=8, color=GREEN)
    axR.set_xlabel("coupling K")
    axR.set_ylabel("fraction in the shared now")
    axR.set_title("the shared now is macroscopic", fontsize=10, style="italic")
    axR.set_ylim(0, 1.0)
    fig.tight_layout()
    fig.savefig(f"{OUT}/fig9_kuramoto.png", bbox_inches="tight")
    plt.close(fig)


# ==========================================================================
# COGNITIVE-SECURITY LAYER  (after Friedman 2026, doi:10.5281/zenodo.20834847)
```

```
# Source-anchored claim ledger + declared evidence ceilings + PASS/FAIL gates.
# Each figure is bound to (i) where in the paper it is claimed, (ii) the finite
# surrogate that stands in for the formal object, (iii) the positive control
# that the intended mechanism is present, (iv) the discriminating test that a
# stronger reading is rejected, and (v) the EVIDENCE CEILING: what must NOT be
# inferred from it.
# ===========================================================================
CLAIM_LEDGER = {
 "fig2": dict(
    anchor="Sec.4 'the rupture is screen-inducing'; eq.(2) C.Omega=1",
    surrogate="bipartite A(x)E pure state; reduced rho_A across saturation s~C.Omega",
    control="POSITIVE: rho_A stays PSD and unit-trace to 4e-16 at every s",
    discriminating="a definite screen EXISTS at rupture -> CRR inherits, not escapes, the impossibility",
    ceiling="Finite-dim illustration that a screen forms. Does NOT measure any real "
            "boundary and does NOT weaken the impossibility theorem."),
 "fig3": dict(
    anchor="Sec.4 'failure against the unitarity requirement'; eq.(3) kernel e^{C/Omega}",
    surrogate="kernel weights e^{C/Omega} over 12 accumulation bins (Z2: Omega=1/pi, C*=pi)",
    control="POSITIVE: peak = e^{pi^2} to machine precision",
    discriminating="literal 'information-pump' reading REJECTED; renormalised reading conserves "
                   "measure yet lowers entropy",
    ceiling="A property of the written kernel, not evidence that any physical regeneration is "
            "non-unitary; it flags an owed free-energy budget, nothing more."),
 "fig4": dict(
    anchor="Sec.4 'the self-measurement obstruction'",
    surrogate="diagonal read-outs of a fixed rho_A in 8 Haar-random measurement frames",
    control="POSITIVE: the eigenbasis read-out equals the true S(rho_A) exactly",
    discriminating="every arbitrary frame OVERSHOOTS the true value; equality only in the eigenbasis",
    ceiling="A finite-dimensional SHADOW of the universal impossibility; it illustrates, does not "
            "prove it, and measures no real agent."),
 "fig6": dict(
    anchor="Sec.6 'the coincidence of map and territory'; eqs.(2)-(3)",
    surrogate="deterministic saturating sawtooth of C.Omega within successive occasions",

    control="POSITIVE: gap 1-C.Omega closes to 0 at each saturation, reopens after",
    discriminating="schematic only -- no stronger reading asserted",
    ceiling="A dynamics of the map-territory gap; not a measurement of any experienced 'now'."),
 "fig7": dict(
    anchor="Sec.7 'retensing the forbidden correlation'",
```

```
        surrogate="qubit + chaotic (GUE) bath; S_A(t)=1/2 I(A:E) vs accumulated Fisher-Rao C(t)",
        control="POSITIVE: de Bruijn identity holds; C and S_A touch only at t=0",
        discriminating="strong identity C=I(A:E) REJECTED; only the bound C>=S_A survives (1920 pts)",
        ceiling="Establishes a bound in a chosen model, not a law of nature; the forbidden correlation "
                "is bounded, never recovered."),
    "fig8": dict(
        anchor="Sec.8 'two systems'; Sec.2.1 doubling (2:1 topological ratio)",
        surrogate="2:1 resonance normal form (rotation number) + coupled-qubit co-determination",
        control="POSITIVE: rho locks at 2.0005 at the topological ratio for any kappa>0",
        discriminating="2:1 tongue separated from GENERIC mode-locking by its WIDTH (0.04 -> 0.80), not "
                       "its mere existence",
        ceiling="A phase-reduced reduction of eqs.(2)-(3); topological centring is a claim about the "
                "model, not a measured entrainment."),
    "fig9": dict(
        anchor="Sec.8 'many systems' (Kuramoto)",
        surrogate="mean-field Kuramoto, Lorentzian frequencies (half-width gamma=1)",
        control="POSITIVE: r(K)=sqrt(1-Kc/K), Kc=2gamma reproduced to 0.015 (< authors' 0.025)",
        discriminating="finite-size scaling: sub-critical r->0 (~N^-0.47), supra-critical N-independent",
        ceiling="Establishes existence and finite-size reality of a transition; the value of Kc depends "
                "on the symmetry-class spectrum CRR does not fix."),
}

# Declared PASS/FAIL tolerance bands for every headline number (a Friedman-style
# artifact gate: the number the paper prints must fall inside its band).
GATES = [
    ("fig2_S_at_rupture_bits",  1.999,  2.001),
    ("fig2_max_negativity",    -1e-9,   1e-9),
    ("fig2_max_trace_error",    0.0,    1e-12),
    ("fig3_peak_kernel",        1.90e4, 1.95e4),    # e^{pi^2}=19333.7
    ("fig3_literal_nats",       9.86,   9.88),      # pi^2
    ("fig3_renorm_dH_nats",    -1.40,  -1.28),
    ("fig4_S_true_bits",        1.62,   1.63),
    ("fig4_eig_read_bits",      1.62,   1.63),       # eigenbasis touches truth
    ("fig7_min_margin",        -1e-9,   1e-6),       # C>=S_A, touches 0 at t=0
    ("fig7_corr_lo",            0.95,   1.0),
    ("fig8_tongue_width_k025",  0.70,   0.8001),
    ("fig8_codet_max",          0.05,   0.30),
    ("fig9_max_val_err",        0.0,    0.025),       # within the authors' own standard
    ("fig9_Kc",                 1.99,   2.01),
```

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    ("fig9_below_exponent",    -0.60,  -0.35),
]

def print_claim_ledger():
    print("\n==============  SOURCE-ANCHORED CLAIM LEDGER  ==============")
    for fig, c in CLAIM_LEDGER.items():
        print(f"[{fig}] anchor : {c['anchor']}")
        print(f"       surrogate     : {c['surrogate']}")
        print(f"       control       : {c['control']}")
        print(f"       discriminating: {c['discriminating']}")
        print(f"       EVIDENCE CEILING: {c['ceiling']}\n")

def figure_hashes():
    """SHA-256 of every generated PNG, so a figure cannot silently drift from
    the number it reports (figure-integrity gate)."""
    hashes = {}
    for fn in sorted(os.listdir(OUT)):
        if fn.startswith("fig") and fn.endswith(".png"):
            with open(os.path.join(OUT, fn), "rb") as f:
                hashes[fn] = hashlib.sha256(f.read()).hexdigest()
    return hashes

def validation_gates():
    """Re-check every headline number against its declared band. Returns
    (all_pass, report)."""
    print("\n==================  VALIDATION GATES  ==================")
    report = {}
    n_pass = 0
    for key, lo, hi in GATES:
        val = RESULTS.get(key, None)
        ok = (val is not None) and (lo <= val <= hi)
        report[key] = dict(value=val, band=[lo, hi], passed=bool(ok))
        n_pass += int(ok)
        print(f"  [{'PASS' if ok else 'FAIL'}] {key:26s} = "
              f"{val!s:<22} band [{lo}, {hi}]")
    all_pass = (n_pass == len(GATES))
    print(f"  ----> {n_pass}/{len(GATES)} gates PASS")
    return all_pass, report


# ==========================================================================
if __name__ == "__main__":
    print("Fig 1 (schematic) ..."); fig1_geometry()
    print("Fig 2 screen induction ..."); fig2_screen_induction()
    print("Fig 3 kernel fork ..."); fig3_kernel_fork()
    print("Fig 4 self-measurement ..."); fig4_self_measurement()
    print("Fig 5 (schematic) ..."); fig5_two_pictures()
    print("Fig 6 every now ..."); fig6_every_now()
```

```python
    print("Fig 7 retensing (bath sims) ..."); fig7_retensing()
    print("Fig 8 two-body ..."); fig8_two_body()
    print("Fig 9 Kuramoto ..."); fig9_kuramoto()

    print("\n================  HEADLINE NUMBERS  ================")
    for k, v in RESULTS.items():
        print(f"{k:32s} : {v}")

    print_claim_ledger()
    all_pass, gate_report = validation_gates()
    hashes = figure_hashes()

    # byte-stable, wall-clock-free results ledger
    out = dict(headline_numbers=RESULTS,
               claim_ledger=CLAIM_LEDGER,
               validation_gates=gate_report,
               all_gates_pass=all_pass,
               figure_sha256=hashes)
    with open(f"{OUT}/results.json", "w") as f:
        json.dump(out, f, indent=2, sort_keys=True)

    print("\nSaved figures + results.json to", OUT)
    print("ALL GATES PASS" if all_pass else "!! SOME GATES FAILED -- inspect above")
```

References

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[3] C. Fields, K. Friston, J. F. Glazebrook, and M. Levin, *A free energy principle for generic quantum systems*, Progress in Biophysics and Molecular Biology **173**, 36–59, 2022.

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