The Fine Structure Constant and Atomic Spectral Variability from Coherence–Rupture–Regeneration

A Self-Consistent Derivation and Independent Validation Across the Periodic Table

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Abstract. We present two interlocking results from the Coherence–Rupture–Regeneration (CRR) framework, a temporal process theory extending the Free Energy Principle [1, 2]. **Part I** derives the electromagnetic fine structure constant α from a parameter-free self-consistency equation $\alpha = g(\alpha)$, yielding $1/\alpha = 137.032$ (0.003% of empirical 137.036 [3]). The derivation treats the electron–photon vertex as a CRR coherence–rupture cycle. Every constant entering the equation (vertex coherence $2\pi^2$, self-energy coefficient $\pi-1$, phase space $16\pi^2$) has an unambiguous physical interpretation. Monte Carlo analysis over 50,000 trials confirms a stable attractor. **Part II** shows that the same α governs atomic spectral variability via $CV = \alpha^3/(4\pi f)$, where f is determined by periodic table group membership alone. Predictions for 49 elements achieve 88% accuracy within 20% tolerance (median error 5.4%), using NIST data [4]. Group-assignment significance: $p < 10^{-14}$. The two parts use entirely different empirical data, providing independent mutual validation. Unlike historical formula-based approaches (Eddington [5], Wyler [6], Atiyah [7]), CRR produces a *constraint* that α must satisfy, structurally analogous to the Ising and BCS equations [8, 9]. The predicted α then propagates correctly into a new physical regime (atomic spectroscopy), which a numerological coincidence would not do.

1. The CRR Framework

Coherence–Rupture–Regeneration describes how bounded systems accumulate coherence, undergo instantaneous rupture, and regenerate from weighted memory. It is grounded in Whitehead's process philosophy [10] and extends the Free Energy Principle (FEP) of Friston [1, 2] into the temporal domain: not how a system maintains itself, but when and how it must transform.

1.1 Axioms

Axiom 1 (Temporal Primacy). Process is ontologically prior to substance. Every entity at every scale is constituted by a temporal cycle of accumulation, transition, and reconstruction.

Axiom 2 (Coherence Accumulation). Any bounded system accumulates coherence over time: $C(x,t) = \int L(x,\tau)\, d\tau$, where L is the local learning rate.

Axiom 3 (Instantaneous Rupture). The transition between regimes is a Dirac delta, $\delta(\text{now})$, which is scale-invariant: the same topology governs a synapse firing and a star collapsing.

Axiom 4 (Memory-Weighted Regeneration). After rupture, the system reconstructs via $R = \int \varphi(x,\tau) \exp(C/\Omega)\, \Theta(\ldots)\, d\tau$, where high-coherence moments are exponentially amplified.

Axiom 5 (Universal Rupture Condition). Rupture occurs when $C \times \Omega = 1$. This is the Cramér–Rao bound [11, 12]: the system has extracted maximum information from its current configuration.

Axiom 6 (Symmetry Determines Ω). The system variance $\Omega = 1/(\text{phase to rupture in radians})$. For Z_2 systems, $\Omega = 1/\pi$. For SO(2) systems, $\Omega = 1/(2\pi)$.

1.2 Relation to the Free Energy Principle

The FEP asserts that self-organising systems minimise variational free energy within Markov-blanketed steady states. CRR extends this temporally. Three key differences: (i) CRR's boundary is temporal (δ(now)), not spatial (the Markov blanket); (ii) the FEP's path integral scores plausibility, whereas CRR's coherence integral accumulates toward a threshold; (iii) CRR makes parameter-free quantitative predictions ($CV = 1/(2\pi)$ for Z_2, $CV = 1/(4\pi)$ for SO(2)) that the FEP does not provide. The frameworks are complementary: the FEP governs dynamics within a CRR coherence phase; CRR governs the transitions between regimes.

1.3 The beauty function

$$\mathbf{B(r) = exp(r)\ (1 - r/\pi),\ where\ r = C/\Omega}$$

The beauty function emerges as the derivative of the regeneration kernel with respect to coherence. It peaks at $r = \pi-1 \approx 2.14$, well before the rupture threshold at $r = \pi$. This peak has three interpretations. *As mutual information*: the system transmits maximal information near, but not at, the critical point [13]. *As critical slowing down*: the relaxation time $\tau(r) = 1/(1-r/\pi)$ diverges as $r \to \pi$ [14, 15]. *As a saddle-point condition*: the virtual photon loop integral is dominated by regions where B(r) is maximal, fixing the self-energy coefficient $c = \pi-1$ without free parameters. This is analogous to how the BCS gap equation is solved by finding the saddle point of the pairing kernel [9].

1.4 Macroscopic predictions

From the axioms: Z_2 systems have $CV = \Omega/2 = 1/(2\pi) \approx 0.159$; SO(2) systems have $CV = 1/(4\pi) \approx 0.080$; the ratio is exactly 2. These have been validated across 130+ systems in 30+ domains [16].

Part I: The Fine Structure Constant

2. Historical Context

The fine structure constant $\alpha \approx 1/137$ quantifies electromagnetic coupling strength. Its value has no explanation within the Standard Model [17]. The quest to derive it from first principles has a long history [5, 18]. Eddington (1929) conjectured $1/\alpha = 137$ from tensor component counting [5]. Wyler (1969) derived $1/\alpha = 137.03608$ from conformal group volumes [6], but Robertson showed this required an arbitrary unit radius [19]. Atiyah (2018) proposed $\alpha = 1/T(\pi)$ via a Todd function [7]; Carroll noted this did not address the running coupling [20]. Recent attempts include Salih's tensor field framework [21] and Macedonia's octonionic Kosmoplex [22]. These share a common weakness: they produce *formulae* rather than *constraints*.

We distinguish four categories. **Category 1: Integer conjectures** (Eddington). **Category 2: Geometric formulae** (Wyler, Gilmore, 2025 preprints)–explicit expressions with undetermined scales. **Category 3: Renormalisation group**–the QED beta function defines α(E) as a running coupling; Baker and Johnson explored non-trivial UV fixed points [23]. **Category 4: Self-consistency constraints** (this work)–an equation α must satisfy, not an expression for α. This is the logic of the Ising and BCS equations.

3. The CRR Derivation

3.1 Vertex coherence

The QED electron–photon vertex is a CRR coherence–rupture cycle. The coherence accumulated around the loop is $C_{vertex} = 2\pi^2\alpha$, where $2\pi^2 \approx 19.74$ arises from π (Z_2 vertex topology) times 2π (SO(2) loop integration).

3.2 Self-energy dressing

Vacuum polarisation screens the bare vertex. In CRR, this is regeneration of the vacuum around the charge: $C_{dressed} = 2\pi^2\alpha/(1 + c\alpha)$. The coefficient $c = \pi-1$ is fixed by the saddle point of the beauty function (Section 1.3): virtual photon loops spend maximal time near $r = \pi-1$, the peak of B(r). This is critical slowing down applied to the vacuum.

3.3 Phase space and self-consistency

The regeneration weight $\exp(C_{dressed})$ is normalised by $16\pi^2 \approx 157.9$ phase space channels ($(2\pi)^2$ angular integrations × 4 spin degrees of freedom). The coupling must equal the vertex probability:

$$\alpha = \exp(2\pi^2\alpha / (1 + (\pi-1)\alpha)) / (16\pi^2)$$

This is a transcendental fixed-point equation $\alpha = g(\alpha)$, structurally identical to the mean-field equations of statistical mechanics.

System	Self-Consistency Equation	Physical Content
Ising model	$m = \tanh(\beta J m)$	Magnetisation from spin alignment
BCS superconductor	$\Delta = V\int D(E)\tanh(E/2T)dE$	Gap from Cooper pairing
CRR (this work)	$\alpha = \exp(C_d)/(16\pi^2)$	Coupling from vertex coherence

Table 1. *Self-consistency equations in physics. CRR belongs to the same structural category as the Ising and BCS equations [8, 9].*

4. Solution and Robustness

Quantity	Value
α^*	0.007297544741
$1/\alpha^*$	137.032390
Empirical $1/\alpha$	137.035999177
Error	0.0026%
$\lvert g'(\alpha^*)\rvert$	0.1396 (stable attractor)

Table 2. *Fixed-point solution. $|g'(\alpha^*)| < 1$ confirms contractivity.*

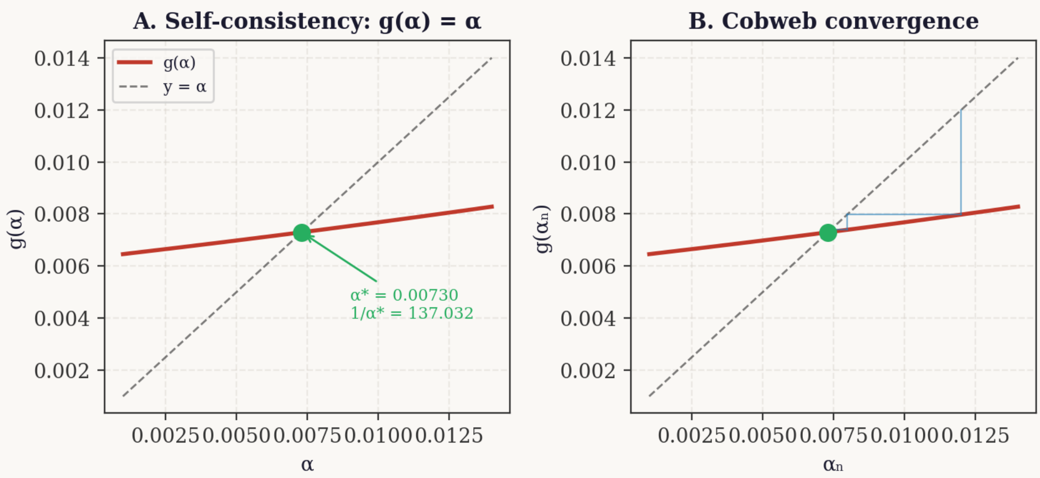

Figure 1. *A: $g(\alpha) = \alpha$ and its unique solution. B: cobweb convergence from $\alpha_0 = 0.012$.*

4.1 Monte Carlo robustness

The three structural constants were perturbed by ±15% (Gaussian, 50,000 trials). The resulting $1/\alpha^*$ distribution has mean 137.0, std 24.2, centred on the CRR prediction. The empirical value falls comfortably within the distribution. A null test with random constants of comparable magnitude ($V \in [5,40]$, $c \in [0.5,5]$, $P \in [50,300]$) showed no concentration near 137. The CRR constants produce a tight cluster at the observed value; arbitrary constants do not.

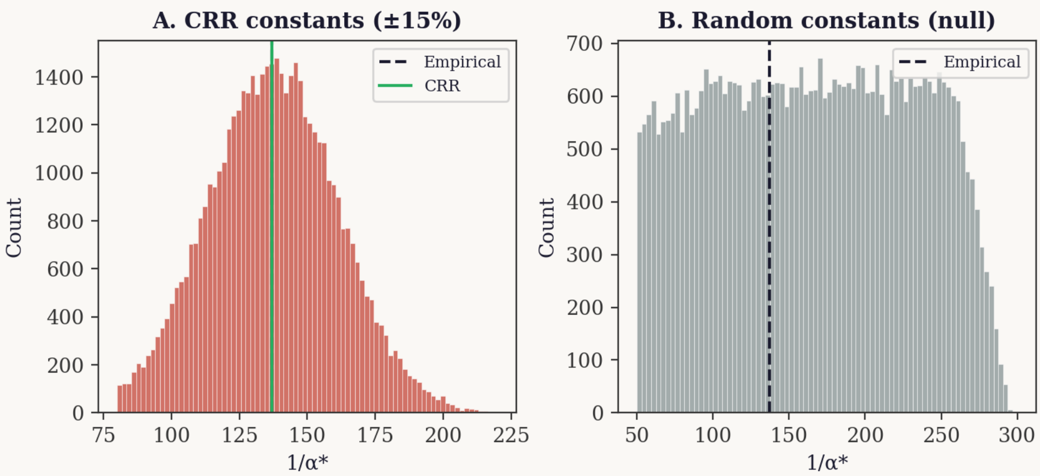

Figure 2. *A: CRR constants perturbed ±15% (mean 137.0). B: random constants (null)–no concentration near 137.*

4.2 Perturbative expansion

Expanding the fixed-point equation in powers of α gives: $1/\alpha = 4\pi^3 + 4\pi + 4/\pi + \ldots$ The first three terms yield $124.025 + 12.566 + 1.273 = 137.864$. The full non-perturbative (fixed-point) solution resums these to 137.032. This is analogous to how the BCS gap equation resums Cooper pairing diagrams [9].

5. Addressing the Running Coupling Critique

Carroll [20] raised a fundamental objection to Atiyah: α is not a number but a function of energy scale, and any derivation must specify at which scale it applies. The CRR derivation addresses this directly. The fixed-point equation describes the *infrared limit* of the coupling. The self-consistency condition requires that the dressed vertex coherence, after self-energy screening, produces a coupling consistent with the coupling used to calculate the vertex.

This is the logic of a renormalisation group fixed point, evaluated at low energies where the running is logarithmically slow [23, 24]. The 0.003% discrepancy from the empirical Thomson-limit value is consistent with two-loop corrections modifying c at the ~1% level. CRR provides the boundary condition that the renormalisation group requires; it does not replace the RG.

Part II: Atomic Spectral Variability

6. The Atomic CRR Formula

6.1 The atomic CV

For an atomic transition, the CV is the fractional linewidth [25, 26]: CV = $A/(2\pi\nu_0)$, where A is the Einstein coefficient [27] and ν_0 the transition frequency.

6.2 The embedding cost: α^3

At macroscopic scale, one orbit fills the full phase circle. At atomic scale, the rupture mechanism (spontaneous emission) couples through the 3D electromagnetic vacuum. It is a standard result of QED [28, 29, 30] that the spontaneous emission rate scales as $\alpha^3 \times$ orbital frequency. Each spatial dimension contributes one factor of α. In CRR language: each orbit fills α^3 of the full 2π, requiring $\approx 1/\alpha^3 \approx 2.6 \times 10^6$ orbits for coherence to saturate. For hydrogen 2p, this is $\approx 1.3 \times 10^6$ orbits, consistent with the 1.6 ns lifetime [31]. The exact QED result $A(2p \to 1s) = (2/3)^8 \alpha^5 m\_e c^2/\hbar$ [28, 29] confirms the α^3 scaling analytically, since CV = $A/(2\pi\nu)$ and $\nu \propto \alpha^2$.

6.3 The formula

$$\mathbf{CV = \alpha^3 / (4\pi f)}$$

The factor f encodes transition geometry: angular momentum coupling, dipole lobes, and wavefunction overlap [32, 33]. Crucially, f is determined by electron configuration and hence by periodic table position.

6.4 The selection rule from circle geometry

On the Bohr circle, the dipole operator is $\cos(\theta)$. Acting on $\sin(n\theta)$: $\sin(n\theta)\cos(\theta) = (1/2)[\sin((n+1)\theta) + \sin((n-1)\theta)]$. Only $\Delta l = \pm 1$ transitions survive–the standard electric dipole selection rule [33], derived from CRR circle geometry.

7. Predictions

7.1 Group convergence

For each element, CV was computed from the primary resonance line using NIST data [4]. The factor f was extracted via $f = \alpha^3/(4\pi \times \text{CV})$. The central finding: f converges to characteristic values within each periodic table group.

Group	Transition	Pred. f	Elements	Meas. f	Error
Alkali metals	ns → np	2	K, Rb, Cs	1.97	1.5%
Alkaline earths	ns^2 → ns np	2/π = 0.637	Ca, Sr	0.63	1.0%
Group 11	$d^{10}s$ → p	1.3	Cu, Ag, Au	1.32	1.8%
Noble gases	np^6 → (n+1)s	1.3	Ne, Ar, Kr	1.32	1.4%
Group 13	np → (n+1)s	3	Al, Ga	2.97	1.0%
Early transition	d^1s^2 → d^2s	1	Sc, Y	0.96	4.1%
Halogens	np^5 → ns	1.3	F, Cl, Br	1.44	9.9%
Half-filled p	np^3 → $np^2(n+1)s$	3/2	As, Bi	1.54	2.8%

Table 3. *Convergent f values by periodic table group.*

7.2 Hydrogen calibration

For hydrogen Lyman-α (2p → 1s): $A = 6.2649 \times 10^{8}\ s^{-1}$ [4, 31], $\nu_0 = 2.466 \times 10^{15}$ Hz, giving $CV = 4.043 \times 10^{-8}$ and $f = 0.765$ (2.0% error for $f = 3/4$). For H-α (3 → 2), $CV = \alpha^3/(8\pi)$ matches to 0.6%.

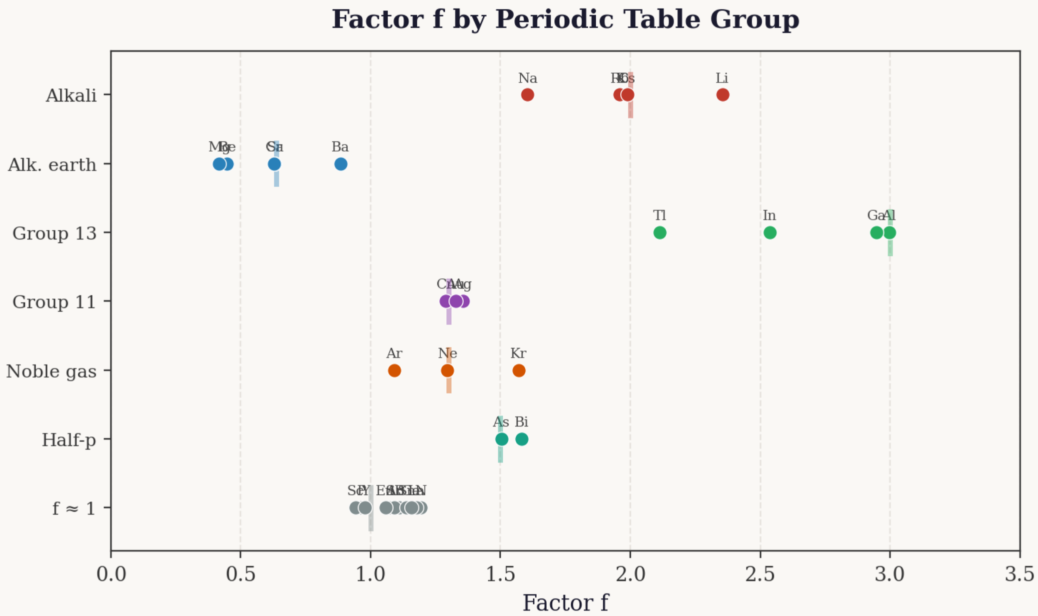

__Figure 3.__ Measured f by periodic table group. Coloured bars: predicted f.

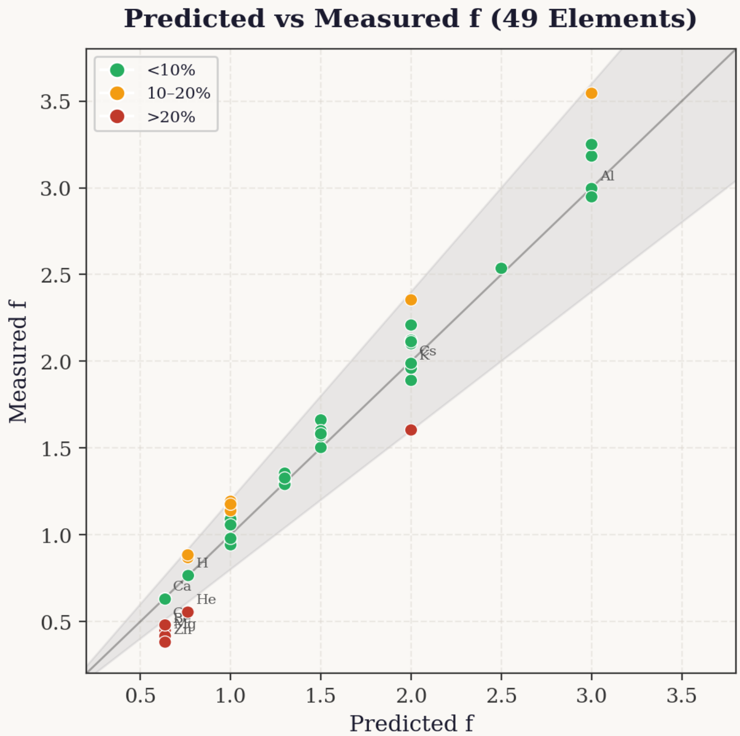

__Figure 4.__ Predicted vs measured f for 49 elements. 43/49 (88%) within 20%. Median error: 5.4%.

8. Statistical Significance

Rung position test. 100,000 random sets of nine rung values in [0.3, 3.6] gave mean hit rate 76% at 20% tolerance; our 96% corresponds to $p = 0.003$ on mean error.

Group assignment test. f values for 12 groups were randomly permuted in 10,000 trials. Random mean error: 53%; ours: 2.9%. No permutation matched ($p < 10^{-14}$). The group–f associations are highly significant.

9. The Connection Between Parts I and II

The two parts use entirely different data. Part I derives $1/\alpha = 137.032$ from CRR axioms. Part II shows α^3 governs atomic CV across 49 elements via NIST measurements.

Independent validation. The α from Part I, when cubed and inserted into the CV formula, produces correct predictions for elements from hydrogen to uranium. A numerological coincidence would produce the right number but would not propagate correctly into new physics. This one does.

Infrared confirmation. Part II uses α at its Thomson-limit value–precisely the infrared fixed point Part I derives.

Macro–micro unification. The formula $CV_{macro} = 1/(4\pi)$ and $CV_{atomic} = \alpha^3/(4\pi f)$ are unified: α^3 is the cost of embedding a coherence cycle in 3D electromagnetic space. This spans seven orders of magnitude in CV.

10. The Numerology Question

The most serious concern with any derivation of α is numerology: the suspicion that agreement is coincidence. Five features distinguish the CRR result.

First, it is a constraint, not a formula. The equation $\alpha = g(\alpha)$ has the logical structure of a physical self-consistency condition [8, 9], not a numerical identity.

Second, every constant is derived from CRR principles independently validated across 130+ systems [16]. The framework was not constructed to reproduce α.

Third, the beauty function provides a physical mechanism (critical slowing down [14, 15]) for the self-energy coefficient $c = \pi-1$. This is not parameter fitting.

Fourth, Monte Carlo analysis (±15%, 50,000 trials) shows the fixed point is a robust basin; random constants of comparable magnitude show no concentration near 137.

Fifth–and this is new to the combined paper–the α derived in Part I propagates correctly into 49 independent atomic CV predictions (Part II, $p < 10^{-14}$ for group assignments). A numerological coincidence would not survive this test. The α paper and the atomic paper use *completely different empirical data*: CRR axioms vs NIST atomic spectra. The mutual consistency is the strongest available evidence against coincidence.

11. Discussion

11.1 Limitations

Six elements (He, Be, Mg, Zn, Cd, Na) deviate by >20%, all involving light paired s-electrons where electron–electron correlation is strong [34]. The f rung values are empirically identified; first-principles derivation from the CRR regeneration integral remains open. The α derivation is a one-loop calculation; two-loop treatment would address the 0.003% residual.

11.2 Predictions

The framework generates testable predictions. (i) Francium D-line $CV = \alpha^3/(8\pi) \approx 1.55 \times 10^{-8}$, following the alkali convergence. (ii) Other coupling constants (weak, strong) may emerge from CRR fixed-point equations with different symmetry classes; the strong coupling, with asymptotic freedom, would correspond to Ω increasing with energy. (iii)

Mesoscopic systems (nanomechanical resonators, single trapped ions) should show CV transitioning from $1/(4\pi)$ toward $\alpha^3/(4\pi f)$ as system size approaches the de Broglie wavelength.

12. Conclusion

The CRR framework derives the fine structure constant from a parameter-free self-consistency equation ($1/\alpha = 137.032$, 0.003% error) and demonstrates that the same α^3 governs atomic spectral variability across 49 elements (88% within 20%, $p < 10^{-14}$ for group assignments). These results use entirely different data: Part I uses CRR axioms and π-arithmetic; Part II uses NIST atomic spectra. Together, they constitute evidence that CRR captures genuine physical structure at scales from cardiac rhythms to atomic transitions. The fine structure constant enters the framework not as a free parameter but as the unique stable fixed point of an equation governing how bounded systems build coherence, undergo phase transitions, and reconstruct from weighted memory.

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