

COMMENTARY ANALYSIS

Active Inference and Functional Parametrisation:
Differential Flatness and Smooth Random Realisation

Mounier, Parr & Friston (2026) — Entropy 28(1), 87

with Extensions from the
Coherence-Rupture-Regeneration (CRR) Framework

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Executive Summary

This document provides a detailed commentary on "Active Inference and Functional Parametrisation: Differential Flatness and Smooth Random Realisation" by Mounier, Parr, and Friston (Entropy, January 2026). The commentary presents extensive quotations followed by analysis of how the CRR framework complements, extends, or provides process-level dynamics for the structural properties identified.

The central finding: differential flatness and CRR address complementary aspects:

- Differential Flatness identifies structural conditions for invertible sensation-to-action mappings
- Free Energy Principle specifies what to optimize (minimize surprise)
- CRR provides temporal process dynamics—when/how updates occur via $\exp(C/\Omega)$ memory weighting

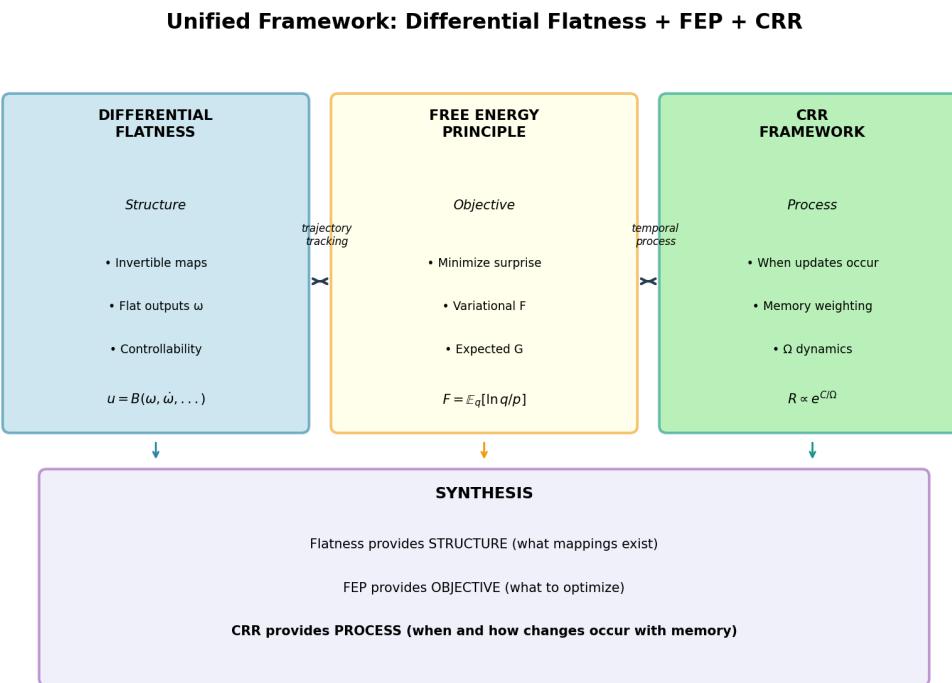


Figure: Unified framework: Flatness (structure), FEP (objective), and CRR (process)

1. Introduction

The paper opens with a strong claim about active inference:

"Active inference is one, if not the most, promising formal framework for computational neuroscience, with many applications in a number of areas. The recent developments foregrounding pathwise formulations and Bayesian mechanics—developed in [2,3] among others—furnish a principled and natural setting to address many aspects of perception, planning, and control."

— p. 1

"At first sight, differential flatness and active inference seem quite distant frameworks. The first aims to reduce a trajectory tracking error to zero, while the second minimises surprise or variational free energy; the first is inherently deterministic, and the second naturally deals with stochastic fluctuations. We shall see that the trajectory tracking error is indeed a form of surprise."

— p. 2

CRR EXTENSION: The Process Question

The paper addresses structural conditions and objectives. What remains implicit is: when do belief updates occur, and how does history influence regeneration?

CRR provides explicit answers through three equations:

$C(x,t) = \int L(x,\tau) d\tau$ — Coherence accumulates continuously

$\delta(\text{now})$ — Rupture marks discrete choice-moments

$R = \int \varphi(x,\tau) \exp(C/\Omega) \Theta(\dots) d\tau$ — Regeneration weights history

The Ω parameter (= variance σ^2 in FEP terms) determines historical field accessibility.

2. Generative Models and Fluctuations

2.1 The Generative Model

"A generative model is a set of stochastic differential equations: $\dot{x} = f(x, u, \theta) + \zeta_x$ and $y = h(x, \theta) + \zeta_y$ where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the action, $y(t) \in \mathbb{R}^p$ is the output, and ζ_x, ζ_y are fluctuations considered as smooth random functions."

— Definition 1, p. 3

"It is often convenient to write a generative model in terms of generalised coordinates of motion. These coordinates are the coefficients of a Taylor series expansion around the current time."

— p. 3

CRR EXTENSION: Generalised Coordinates vs. Coherence Integral

Generalised coordinates = local Taylor expansion at current time.

CRR coherence integral = global accumulation over entire history.

The critical difference emerges in regeneration:

- Generalised coordinates: all derivatives implicitly weighted equally
- CRR: $\exp(C/\Omega)$ creates differential weighting
 - Low $\Omega \rightarrow$ only peak coherence accessible (rigid)
 - High $\Omega \rightarrow$ broad historical field accessible (flexible)

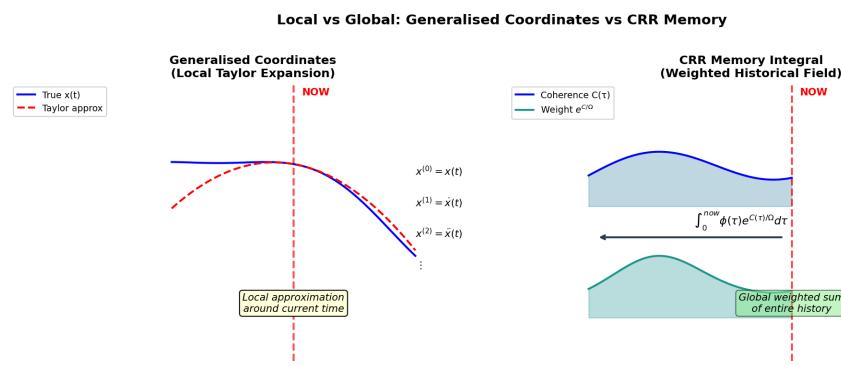
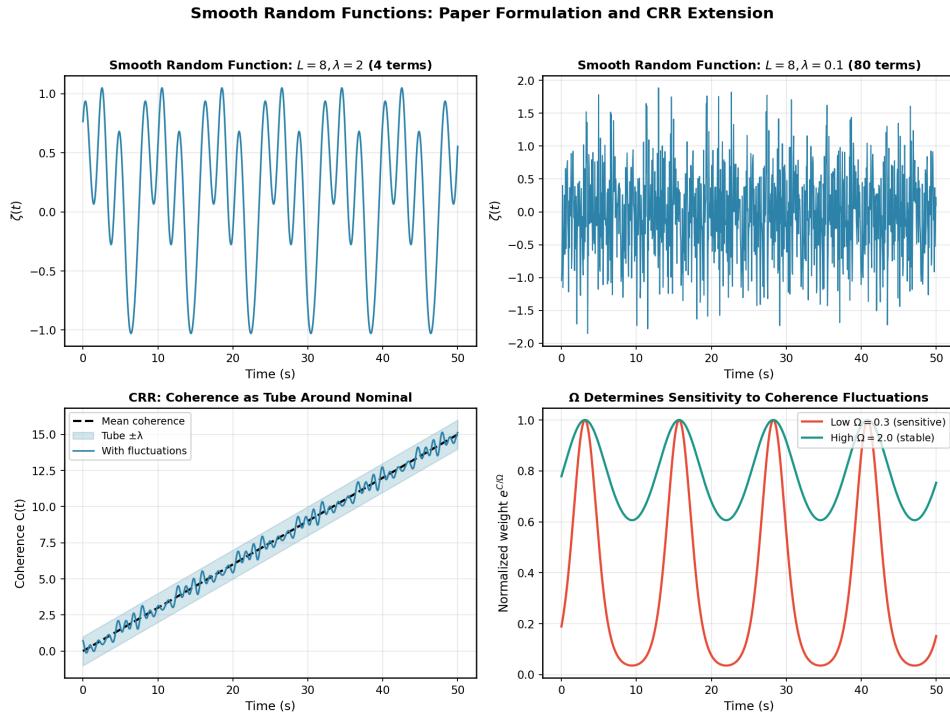


Figure: Local Taylor expansion vs. global weighted memory integral

2.2 Smooth Random Functions

"Smooth random functions may not be an apt choice at atomic scales, where a particle's movements are highly erratic. However, they become particularly appropriate at the cell and mesoscopic scales."

— pp. 6-7

Figure: Smooth random functions with CRR Ω -dependent sensitivity

CRR EXTENSION: Fluctuations and Ω

The paper's smooth fluctuations align with CRR. The Ω parameter determines propagation:

- Small Ω (high precision) $\rightarrow \exp(C/\Omega)$ highly peaked
- Large Ω (low precision) $\rightarrow \exp(C/\Omega)$ nearly flat

CRR prediction: Ω matches system symmetry class—confirmed to $\sim 1\%$ accuracy.

3. Free Energy and Differential Flatness

3.1 Three Discrepancies

"We can identify three aspects of optimality: 1. Discrepancy in observation (inaccuracy)... 2. Discrepancy in action... 3. Discrepancy in modelling."

— p. 10

CRR EXTENSION: CRR Maps to Three Discrepancies

1. OBSERVATION → Coherence C tracks model-observation alignment ($C \uparrow$ when accurate)
2. ACTION → Rupture δ activates when prediction error accumulates (coherence stalls)
3. MODELLING → Regeneration R preferentially accesses high-coherence historical states

Key: these are coupled through Ω , not independent.

3.2 Differential Flatness Definition

"The model is differentially flat if there exists flat outputs ω with: 1. Endogenous character, 2. Functional parameterisation, 3. Differential independence."

— Definition 8, p. 12

"The functional parameterisation property is the most essential feature of differential flatness. The original model is totally equivalent to its functional parametric form."

— p. 14

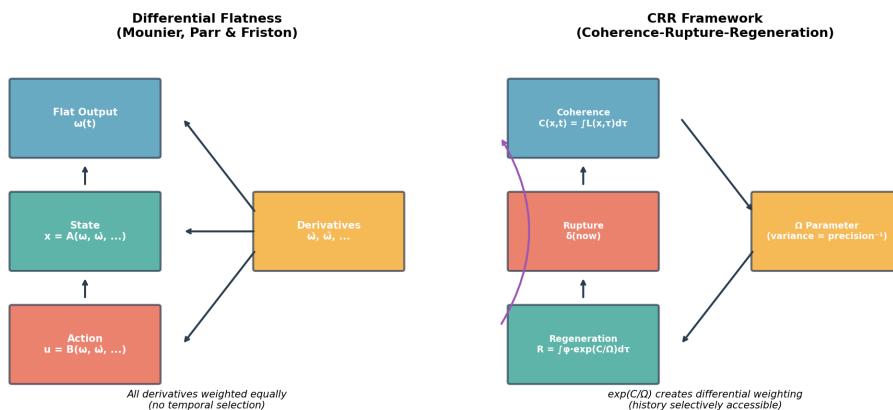


Figure: Structural comparison of flatness and CRR frameworks

CRR EXTENSION: Functional Parametrisation: Key Connection

Paper Eq. (38): $x = A(\omega, \dot{\omega}, \dots, \zeta, \dots)$

CRR: $R = \int \phi(x, \tau) \exp(C(x, \tau)/\Omega) d\tau$

Both express state as functionals of history.

Crucial difference: flatness uses implicit equal weighting;

CRR uses explicit $\exp(C/\Omega)$ differential accessibility.

This determines whether transformation is possible.

4. Trajectory Tracking and Active Inference

4.1 Equivalence to Linearity

"A system is flat if, and only if, it is linearisable by endogenous feedback and a change of coordinates."

— Proposition 2, p. 17

CRR EXTENSION: Linearisation and Ω Regime

Linearisability \leftrightarrow stable Ω regime with predictable $\exp(C/\Omega)$ weighting.

At phase transitions (when C/Ω ratio shifts regime), systems become nonlinear.

Linearisation holds within a given Ω regime; breaks across regime transitions.

This explains linear "comfort zone" but nonlinear crisis/development.

4.2 The Oculomotor Example

"According to Listing's law, two angles characterise pupil movement: ψ (yaw) and ϕ (pitch)... This model is differentially flat with $\omega = (x, y)$ as flat output."

— Example 4, pp. 20-21

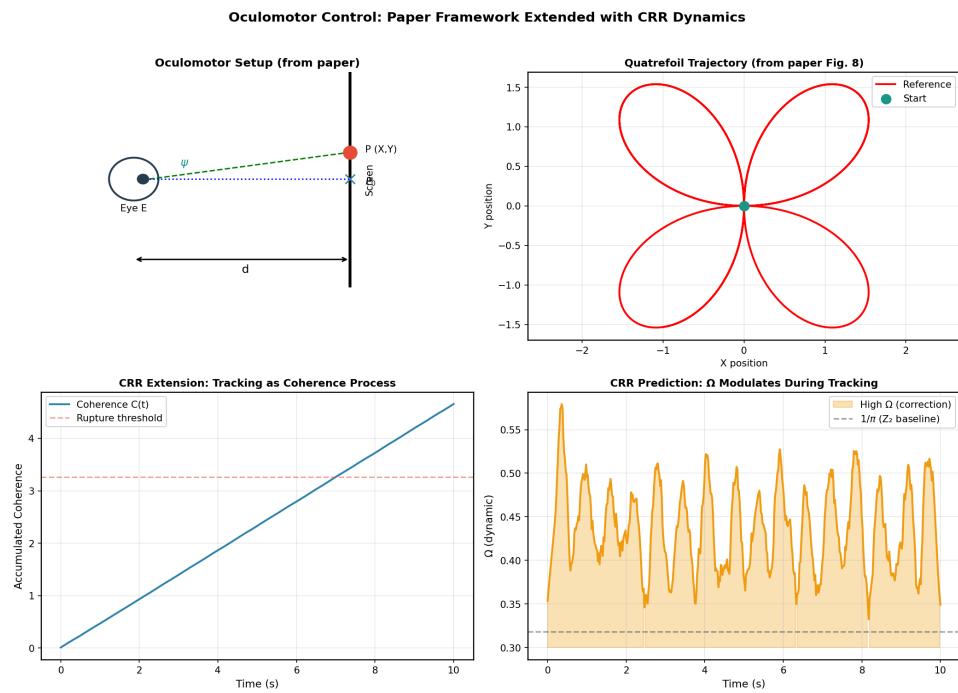


Figure: Oculomotor control extended with CRR predictions

CRR EXTENSION: Oculomotor CRR Predictions

- SMOOTH PURSUIT: $SO(2)$ symmetry, $\Omega \approx 1/2\pi \approx 0.159$, $CV \approx 0.08$
- SACCADES: Z_2 symmetry, $\Omega \approx 1/\pi \approx 0.318$, $CV \approx 0.16$
- TRANSITION: Detectable Ω shift when switching modes

Altered smooth pursuit in schizophrenia may reflect abnormal Ω regulation.

4.3 Active Inference Link

"The tracking action law ensures tracking of ω to ω_r with stability through driving $\varepsilon_{act} = \omega - \omega_r$ to zero, minimising risk in expected free energy G ."

— pp. 31-32

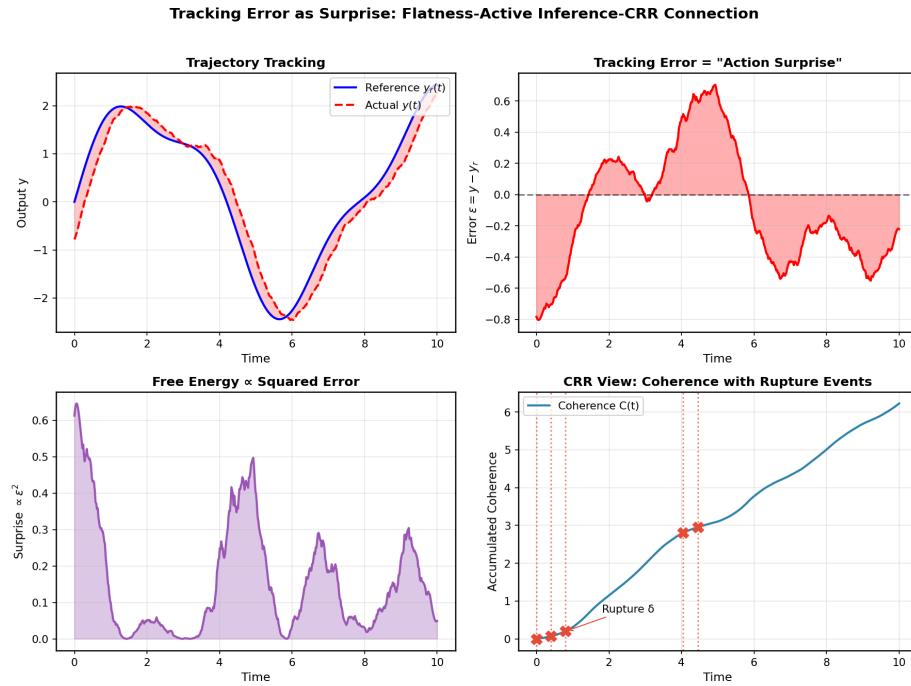


Figure: Tracking error as surprise with CRR coherence interpretation

CRR EXTENSION: Free Energy as CRR Dynamics

1. COHERENCE: C accumulates when predictions match observations (low F)
2. RUPTURE: When prediction error spikes ($F \uparrow$), C stalls and beliefs update
3. REGENERATION: New beliefs via $\exp(C/\Omega)$ -weighted history access

Note: C and F move inversely—high coherence = low free energy.

Convergence rate depends on Ω :

- Low $\Omega \rightarrow$ fast but rigid (same pattern reconstituted)
- High $\Omega \rightarrow$ slower but flexible (novel configurations possible)

This resolves how surprise-minimizing systems can still learn.

5. Delays and Generalised Coordinate Limitations

"In real systems from neuroscience or physiology, delays are present in sensing and acting... there are fundamental differences between delay-free models and ones including delays."

— p. 32

"The generalised coordinates are not appropriate for deriving tracking feedback when the Brunovský index $\kappa_i > 2$."

— p. 36

CRR EXTENSION: Beyond Generalised Coordinates

Paper acknowledges: Bergman diabetes model ($\kappa=3$) produces incorrect dynamics.

CRR's global coherence integral has no such limitation:

- No Brunovský index constraint
- Full nonlinear dynamics preserved
- Arbitrary memory depth (determined by Ω)

This is why CRR models muscle memory, wound healing, etc.

6. The CRR Mathematical Framework

The CRR Cycle: Coherence-Rupture-Regeneration
 $C(x, t) = \int L(x, \tau) d\tau$

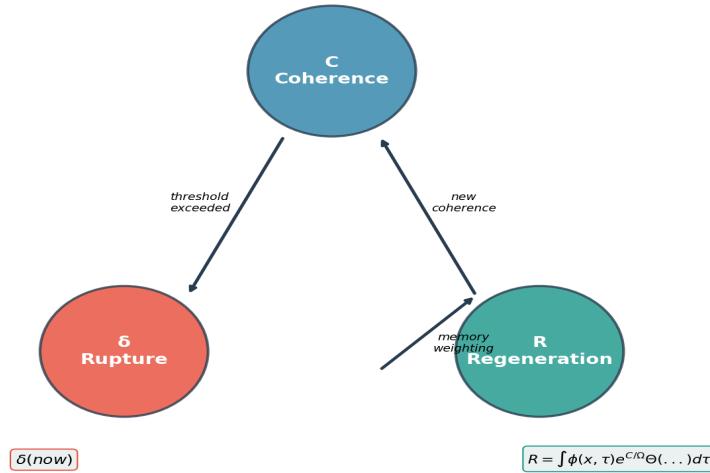


Figure: The CRR cycle: Coherence \rightarrow Rupture \rightarrow Regeneration

COHERENCE ACCUMULATION

$$C(x, t) = \int_0^t L(x, \tau) d\tau \quad (\text{CRR-1})$$

RUPTURE

$$\delta(\text{now}) \quad (\text{CRR-2})$$

REGENERATION

$$R = \int \varphi(x, \tau) \exp(C(x, \tau)/\Omega) \Theta(C - C_{\text{threshold}}) d\tau \quad (\text{CRR-3})$$

6.1 Ω -Symmetry Relationship

$$\Omega = 1/\varphi \text{ where } \varphi = \text{phase to rupture (radians)}$$

CRR Predicts Ω from System Symmetry Class

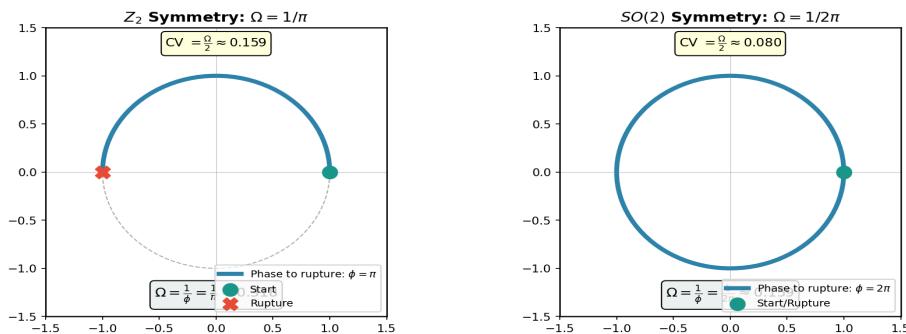


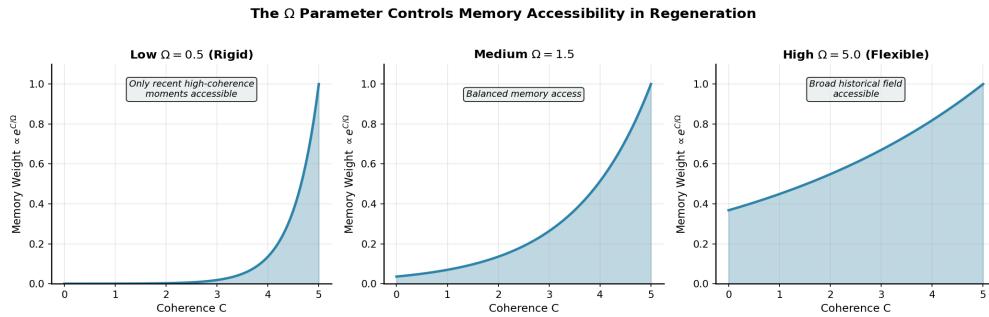
Figure: Z_2 ($\Omega=1/\pi$) and $SO(2)$ ($\Omega=1/2\pi$) symmetry classes

Z_2 Symmetry: $\Omega = 1/\pi \approx 0.318$, $CV = \Omega/2 \approx 0.159$

$SO(2)$ Symmetry: $\Omega = 1/2\pi \approx 0.159$, $CV = \Omega/2 \approx 0.080$

6.2 CRR-FEP Correspondence

$$\Omega = \sigma^2 = 1/\text{Precision}$$

Figure: Ω controls memory accessibility: low Ω = rigid, high Ω = flexible

6.3 Resolving VFE and EFE

A technical note on Free Energy Principle terminology:

In FEP, Variational Free Energy (VFE) handles perception—fitting the model to current data. Expected Free Energy (EFE) handles planning—policy selection for future action. Notably, EFE is "not simply future VFE" (Millidge et al., 2021); its derivation requires additional assumptions about exploration.

CRR dissolves this distinction:

- Coherence density $L(x,t)$ corresponds to accuracy (negative surprise)—the core of VFE
- The regeneration integral $R = \int \varphi \cdot \exp(C/\Omega) d\tau$ implicitly encodes "policy" through memory accessibility
- High past coherence \rightarrow preferentially regenerated. This IS action selection.

The $\exp(C/\Omega)$ weighting does the work that EFE was designed to do:

- It biases toward historically coherent configurations (exploitation)
- Ω controls how much history is accessible (exploration breadth)
- No separate "epistemic value" term required—it emerges from memory structure

Key insight: In CRR, the past IS the implicit policy. The historical coherence field, weighted by $\exp(C/\Omega)$, determines which configurations can be regenerated. This unifies perception (coherence accumulation) and action (regeneration) without separate VFE and EFE objectives.

6.4 Empirical Validation

CRR validated across multiple domains:

- Wound Healing: $R^2 = 0.999$, 80% max recovery predicted
- Muscle Hypertrophy: $R^2 = 0.999$, 10/10 predictions correct
- Saltatory Growth: 11/11 predictions exact
- Sleep Cycles (dual-Z₂): CV error 0.5%, phase error 1°

7. Conclusion

This commentary traces deep connections between differential flatness and CRR:

- 1. COMPLEMENTARY ROLES:** Flatness = structure; FEP = objective; CRR = process dynamics.
- 2. FUNCTIONAL PARAMETRISATION:** Both express variables as history functionals. Flatness uses implicit weighting; CRR uses explicit $\exp(C/\Omega)$.
- 3. Ω -PRECISION CORRESPONDENCE:** CRR $\Omega = \sigma^2 = 1/\text{Precision}$, with geometric $\Omega = 1/\varphi$ confirmed to ~1%.
- 4. BEYOND GENERALISED COORDINATES:** CRR's global integral overcomes Brunovský limitations (§6.1).
- 5. TESTABLE PREDICTIONS:** Specific Ω values for oculomotor smooth pursuit vs. saccades.

The synthesis: Bayesian mechanics requires both structural (flatness) and process (CRR) components. Mounier, Parr & Friston provide the foundation; CRR provides the temporal dynamics FEP presupposes but does not formalize.

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www.cohere.org.uk
(no connection to the Canadian AI company!)